CHAPTER 1

Motion in a Straight Line

INTRODUCTION

A body is at rest when it does not change its position with time and is in motion if it changes its position with time in the frame of reference of the observer.

All motion is relative. There is no meaning of rest or motion without reference to the observer.

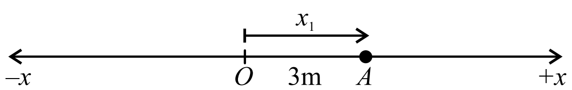
A passenger in a moving train is at rest with respect to another passenger in the same train while both are in motion with respect to observer on the ground. Therefore nothing is at absolute rest or in absolute motion.

To describe the motion of a particle, we introduce four important quantities namely position, displacement, velocity and acceleration. In general motion of a particle in three dimensions these quantities are vectors which have direction as well as magnitude. But for a particle moving in a straight line, there are only two directions, distinguished by designating one as positive and other as negative.

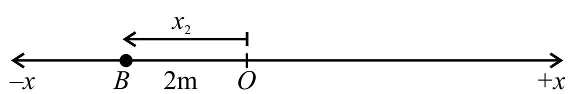
DISTANCE AND DISPLACEMENT

Position

The position of a particle is the location of particle measured with respect to some reference point. It is a vector quantity.



Position of A wrt O, $x_1 = 3$ m



Position of B wrt O, $x_2 = -2$ m

If particle lies towards +ve side of the chosen reference, then its position is also +ve and vice-versa.

Distance

The length of the actual path between initial and final positions of a particle is called distance covered by the particle. It is the actual length of the path covered by the body.

Characteristics of Distance

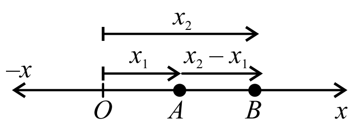
- ❖ It is a scalar quantity
- ❖ It depends on the path
- ❖ It never reduces with time
- ❖ Distance covered by a particle is always positive or zero and can never be negative
- ❖ Dimension: $[M^0 L^1 T^0]$
- ❖ Unit: In CGS: centimeter (cm), In SI system: meter (m).

Displacement

The shortest distance from the initial position to the final position of the particle is called displacement. The displacement of a particle is measured as the change in the position of the particle in a particular direction over a given time interval. It depends only on final and initial positions.

Position of A w.r.t. $O = OA = x_1$

Position of B w.r.t. $O = OB = x_2$



Displacement $= AB = x_2 - x_1$

The direction of displacement here is from A to B. If direction of displacement is along +ve direction of the chosen reference then displacement is +ve and vice versa.

Characteristics of Displacement

- ❖ It is a vector quantity.
- ❖ The displacement of a particle between any two points is equal to the shortest distance between them.
- ❖ The displacement of an object in a given time interval may be +ve, –ve or zero.
- ❖ The actual distance travelled by a particle in the given interval of time is always equal to or greater than the magnitude of the displacement and in no case, it is less than the magnitude of the displacement, i.e. Distance $\geq$ |Displacement|
- ❖ Dimension: $[M^0 L^1 T^0]$
- ❖ Unit: In CGS: centimeter (cm), In SI: system: meter (m).

Note: Distance is always positive but displacement may be +ve, – ve or zero.

AVERAGE SPEED AND INSTANTANEOUS SPEED

Average speed is the ratio of total distance covered by a particle in a given time interval divided by the time interval.

$$v_{avg} = \frac{\Delta s}{\Delta t} \qquad \text{(where } \Delta t = t_2 - t_1\text{)}$$

Instantaneous speed at any instant t is

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- The slope of the distance-time graph provides the value of instantaneous speed.
- The average speed is defined for a time interval while the instantaneous speed is defined at an instant. The word speed normally implies instantaneous speed.
- Average speed and instantaneous speed both are scalar quantities.
- For any moving object, the average speed can never be zero or negative, i.e., $v_{av} > 0$, as total distance covered is always +ve only.
- If a particle travels distances s_1, s_2, s_3, ..., etc., at different speeds v_1, v_2, v_3, ... etc., respectively, then

  $$v_{av} = \frac{\Delta s}{\Delta t} = \frac{\Sigma s_i}{\Sigma(s_i / v_i)}$$

  If $s_1 = s_2 = ... = s_n = s$,

  Then $\frac{1}{v_{av.}} = \frac{1}{n}\left[\frac{1}{v_1} + \frac{1}{v_2} + ...\right] = \frac{1}{n}\Sigma\frac{1}{v_i}$

  Special case: If a particle moves a distance at speed v_1 and comes back to initial position with speed v_2, then

  $$v_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

- If a particle travels at speeds v_1, v_2, ..., etc., for time intervals t_1, t_2, ..., then

  $$v_{av} = \frac{\Delta s}{\Delta t} = \frac{v_1t_1 + v_2t_2 + ... + v_nt_n}{t_1 + t_2 + ...t_n} = \frac{\Sigma v_it_i}{\Sigma t_i}$$

  Special case: If a particle moves for two equal intervals of time at different speeds, then $v_{av} = \frac{v_1 + v_2}{2}$.

AVERAGE VELOCITY

The average velocity is the ratio of displacement from time t_1 to t_2 and the time interval $t_2 - t_1$.

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

INSTANTANEOUS VELOCITY

The instantaneous velocity at any instant t is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The magnitude of instantaneous velocity is always equal to the instantaneous speed.

Instantaneous speed $= |\vec{v}| = \lim_{\Delta t \to 0} \frac{|\Delta x|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

Instantaneous velocity is called simply velocity.

If an object moves along a straight line without changing its direction, then the magnitude of the average velocity is equal to the average speed, otherwise magnitude of average velocity < average speed.

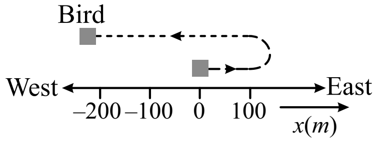
- Velocity can be +*ve* or –*ve* as it is a vector but speed can never be negative as it is the magnitude of velocity.
- If a body is moving with a constant velocity, then the average velocity and instantaneous velocity are equal.
- The velocity of a body is uniform, if both magnitude and direction do not change.
- If a body moves with non-uniform velocity, then magnitude of velocity or direction of velocity may change or both.
- A body can have non-zero speed and zero average velocity when a body completes one revolution around a circle. The average velocity is zero since the displacement is zero. But the average speed is not zero since the distance travelled $\neq 0$.
- If a body is moving with constant speed then its velocity may or may not be constant. In case of uniform circular motion though speed remains constant but velocity changes instant to instant because of change in direction.

Train Your Brain

Example 1: A bird flies towards east at 10 m/s for 100 m. It then turns around and flies at 20 m/s for 15s. Find

(*a*) Its average speed (*b*) Its average velocity

Sol. Let us take the x axis to point eastwards A sketch of the path is shown in the figure. To find the required quantities, we need the total time interval. The first part of the journey took



Δt_1 = (100 m)/(10 m/s) = 10s, and we are given t_2 = 15s for the second part. Hence the total time interval is $\Delta t = \Delta t_1 + \Delta t_2 = 25$ s

The bird flies 100 m east and then (20 m/s) × (15s) = 300 m west.

(*a*) Average speed = $\dfrac{\text{Distance}}{\Delta t}$

$$= \frac{100\,\text{m} + 300\,\text{m}}{25\text{s}} = 16\,\text{m/s}$$

(*b*) The net displacement is

$\Delta x = -200$ m

So, $v_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{-200\text{m}}{25\text{s}} = -8$ m/s.

The negative sign means that v_{av} is directed toward the west.

Example 2: A particle moves with speed v_1 along a particular direction. After some time it turns back and reaches the starting point again travelling with speed v_2. Find (for the whole journey).

(*a*) Average velocity (*b*) Average speed

Sol. (*a*) Since the particle reaches the starting point again, its displacement is zero.

$$\therefore \text{Average velocity} = \frac{\text{Net displacement}}{\text{total time}} = 0$$

(*b*) Let it travel distance *x* while moving away and distance *x* while moving towards the starting point.

Time taken to go away is $t_1 = \dfrac{x}{v_1}$

Time taken while return journey $t_2 = \dfrac{x}{v_2}$

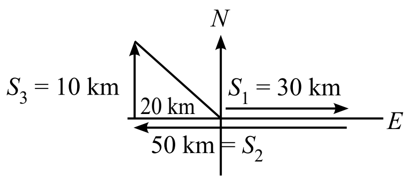
$$\therefore \text{Average speed} = \frac{x + x}{t_1 + t_2} = \frac{2x}{\dfrac{x}{v_1} + \dfrac{x}{v_2}}$$

$$v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

i.e., harmonic mean of individual speeds.

Example 3: A person goes 30 km east, then he walks 50 km west and then he goes 10 km north. Find average speed and average velocity for the whole journey in 15 hrs.

Sol.



$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{30 + 50 + 10}{15}$$

$$= 6 \text{ km/hr}$$

$$\text{Average velocity} = \frac{\text{Total Displacement}}{\text{Total Time}}$$

$$= \frac{\sqrt{(20^2 + 10^2)}}{15} = \frac{10\sqrt{5}}{15}$$

$$\text{Average velocity} = \frac{2\sqrt{5}}{3}\text{km/hr north of west}$$

Concept Application

1. A car moves 30 km with 20 km/hr and then 30 km with 30 km/hr. Find average speed and average velocity for the whole journey in the same straight line.
2. A person goes 20 km *N*, then 20 km *E* and then 20 km *N-E*, find average speed and average velocity if total time taken is 6 hr.

ACCELERATION

The rate of change of velocity of an object with time is called acceleration of the object.

Let *v* and *v*′ be the velocity of the object at time *t* and *t*′ respectively, then acceleration of the body is given by

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- Acceleration is a vector quantity.
- **Unit:** In SI system: m/s^2

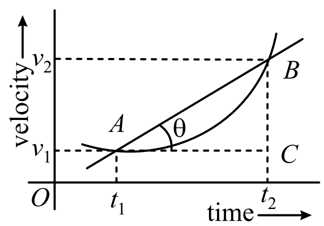
  In CGS system: cm/s^2
- **Dimension :** $[M^0L^1T^{-2}]$

Average Acceleration

When an object is moving with a variable acceleration in a straight line, then the average acceleration of the object for the given motion is defined as the ratio of the change in velocity of the object during motion to the time taken i.e.,

$$\text{Average Acceleration} = \frac{\text{change in velocity}}{\text{total time taken}}$$

Suppose the velocity of a particle is v_1 at time t_1 and v_2 at time t_2.



Then, Change in velocity = $v_2 - v_1 = \Delta v$

Elapsed time in changing the velocity = $t_2 - t_1 = \Delta t$

Thus, $a_{av} = \dfrac{v_2 - v_1}{t_2 - t_1} = \dfrac{\Delta v}{\Delta t}$

$\Rightarrow a_{av} = \dfrac{BC}{AC}$ = tan θ = the slope of chord of $v - t$ graph is average acceleration.

Instantaneous Acceleration

The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration.

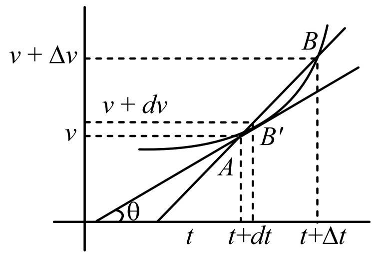
Suppose the velocity of a particle at time $t_1 = t$ is $v_1 = v$ and becomes $v_2 = v + \Delta v$ at time $t_2 = t + \Delta t$,

Then, $a_{av} = \dfrac{\Delta v}{\Delta t}$

If Δt approaches to zero, then the rate of change of velocity will be instantaneous acceleration.

Instantaneous acceleration $a_{\text{inst}} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

Instantaneous acceleration at a point is equal to slope of tangent at that point on velocity time graph in the graph shown.



Slope $= \dfrac{dv}{dt}$ = acceleration

As $v = \dfrac{dx}{dt}$ therefore $a = \dfrac{d}{dt}\left(\dfrac{dx}{dt}\right) = \dfrac{d^2x}{dt^2}$

Thus, instantaneous acceleration of an object is equal to the second derivative of the position w.r.t. time of the object at the given instant.

Note:

(*i*) It is not essential that when velocity is zero acceleration must be zero.

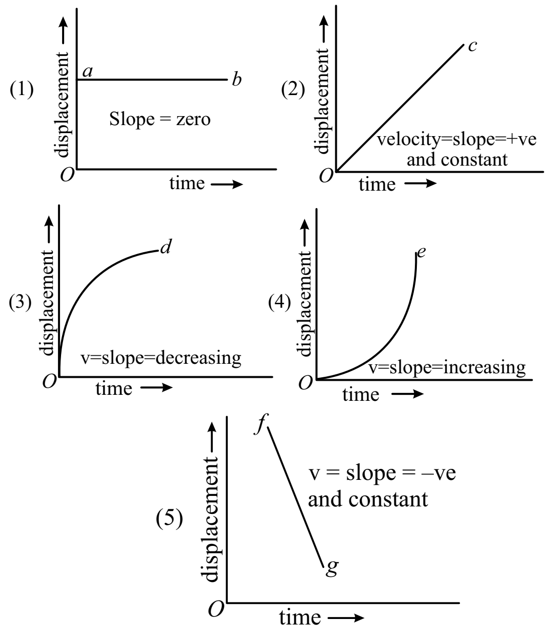
e.g. In vertical motion under gravity at the top point $v = 0$ but $a \neq 0$

(*ii*) If a and v are both positive or both negative, speed of a body increases. If a and v have opposite signs then speed decreases, this is called retardation.

DISPLACEMENT-TIME GRAPHS AND THEIR CHARACTERISTICS

If the displacement-time graph is:

- ❖ A straight line parallel to time-axis, shown by line *ab*, it means that the body is at rest, i.e., $v = 0$.
- ❖ A straight line inclined to time-axis (such as *Oc* and *fg*) shows that body is moving with a constant velocity.
- ❖ A straight line inclined to time-axis by an angle > 90° (line *f g*) represent negative velocity.
- ❖ Of the type of curve *Od* (graph-3) whose slope decreases with time, the velocity goes on decreasing, i.e., motion is retarded.
- ❖ Of the type of curve *Oe* (graph-4) whose slope increases with time, the velocity goes on increasing, i.e., motion is accelerated.



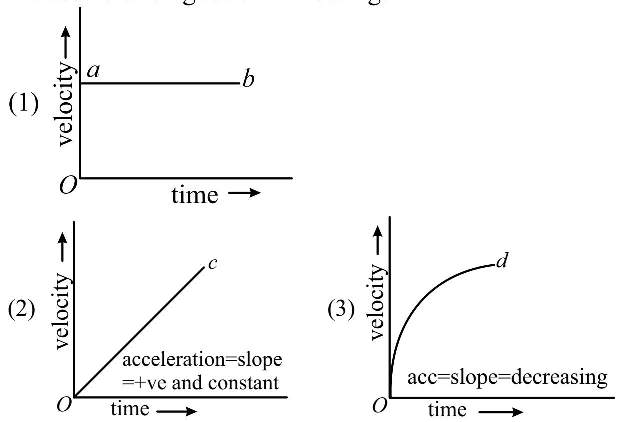
No line can ever be perpendicular to the time axis because it implies infinite velocity.

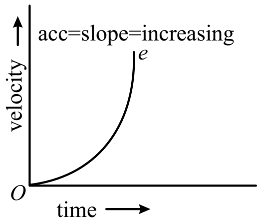
Slope of displacement-time graph at a point gives velocity of the body at that instant.

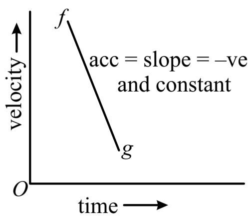
VELOCITY-TIME GRAPHS AND THEIR CHARACTERISTICS

If the velocity-time graph is:

- ❖ A straight line parallel to time axis shown by line *ab*, it means that the body is moving with a constant velocity or acceleration (*a*) is zero.
- ❖ A straight line inclined to the time-axis with +*ve* slope (line *Oc*) it means that the body is moving with constant positive acceleration.
- ❖ A straight line inclined to time-axis with negative slope (line *fg*) it means that the body is moving with constant negative acceleration.
- ❖ A curve like *Od* (graph 3) whose slope decreases with time, the acceleration goes on decreasing.
- ❖ A curve like *Oe* (graph 4) whose slope increases with time, the acceleration goes on increasing.



(4) 

(5) 

Note:

1. No velocity-time graph can ever be perpendicular to the time-axis because it implies infinite acceleration.
2. The area of velocity-time graph with time axis represents the displacement of that body.
3. Slope of velocity time graph at an instant gives the acceleration of the body at that instant.

MOTION WITH CONSTANT ACCELERATION

In different types of motions, the acceleration is either constant or approximately so. For example, near the surface of earth all objects fall vertically with constant acceleration if air resistance is neglected. Even when acceleration is not constant, we can learn something about the motion of the body by using constant acceleration results to be developed later in this section.

Let the velocity of body at $t = 0$ is u and it moves with constant acceleration a and acquires velocity v at time t.

$$\frac{dv}{dt} = a \quad \text{or} \quad dv = adt$$

$$\Rightarrow \int_u^v dv = \int_0^t adt = a\int_0^t dt$$

$$\Rightarrow v\Big|_u^v = at\Big|_0^t$$

$$\Rightarrow v - u = at \quad \text{or} \quad v = u + at \qquad ...(i)$$

To find the displacement, we again integrate.

Let body be at x_0 at $t = 0$ and reaches x at time t

$v = u + at$

$$\frac{dx}{dt} = u + at$$

or $dx = (u + at)dt$

$$\int_{x_0}^x dx = \int_0^t (u+at)dt = u\int_0^t dt + a\int_0^t tdt$$

$$x\Big|_{x_0}^x = ut\Big|_0^t + a\frac{t^2}{2}\Big|_0^t$$

$$x - x_0 = ut + \frac{1}{2}at^2$$

$$S = x - x_0 = ut + \frac{1}{2}at^2 \qquad ...(ii)$$

We can also find, relation between velocity and displacement. By using chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = \frac{vdv}{dx}$$

$$\Rightarrow vdv = adx$$

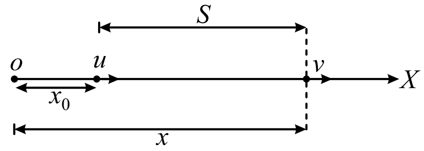
$$\int_u^v vdv = a\int_{x_0}^x dx$$

$$\frac{v^2}{2}\Big|_u^v = a\,x\Big|_{x_0}^x$$

$$\frac{v^2}{2} - \frac{u^2}{2} = a(x - x_0)$$

$$\Rightarrow v^2 - u^2 = 2aS \qquad ...(iii)$$

These relation are very helpful in solving the problems of motion in one dimension. All these relations are given in table below for easy reference.



Equation	Contains		
	s	v	t
$v = u + at$	No	Yes	Yes
$s = ut + \frac{1}{2}at^2$	Yes	No	Yes
$v^2 - u^2 = 2as$	Yes	Yes	No

In simple problems of uniformly accelerated motion, two parameters are given and third is to be found. Depending on convenience one can choose any one of the three relations. The following two relations are also helpful in solving problems.

Displacement of the Body in the n^{th} Second:

$$S_n = S(\text{at } t = n) - S(\text{at } t = n - 1)$$

$$= \left(un + \frac{1}{2}an^2\right) - \left(u(n-1) + \frac{1}{2}a(n-1)^2\right) = u + \frac{a}{2}(2n-1)$$

Average velocity:

$$V_{avg} = \frac{S}{t} = \frac{ut + \frac{1}{2}at^2}{t} = u + \frac{at}{2}$$

$$= u + \frac{v-u}{2} = \frac{u+v}{2}$$

$$\text{or} \quad S = \left(\frac{u+v}{2}\right)t$$

This relation is only valid for uniform acceleration.

Note:

- These equations can be applied only when acceleration is constant.
- If a body moves with uniform acceleration and velocity changes from u to v in a time interval, then average velocity = $\frac{v+u}{2}$.
- If a body moving with uniform acceleration has velocities u and v at two points in its path, then the velocity at the midpoint of given two points = $\sqrt{\frac{u^2+v^2}{2}}$.

- ❖ In position time graph, slope is equal to velocity.
- ❖ In velocity time graph area under the curve is displacement and slope is equal to acceleration.
- ❖ In acceleration time graph area under the curve is equal to change in velocity.
- ❖ **For a body starting from rest and moving with uniform acceleration,**
  - (*a*) The ratio of distances covered in first one sec, two sec, three sec, ... is :

    $1^2 : 2^2 : 3^2 : ...$, i.e., 1 : 4 : 9 : ...

    Ratio of distances covered in

    1st, 2nd, 3rd sec, ... is 1 : 3 : 5 : ...
  - (*b*) The ratio of velocities after

    1 sec, 2 sec, 3 sec, ... is 1 : 2 : 3 : ...

Train Your Brain

Example 4: The displacement of a particle, moving in a straight line, is given by $S = 2t^2 + 2t + 4$ where S is in metres and t in seconds. The acceleration of the particle is

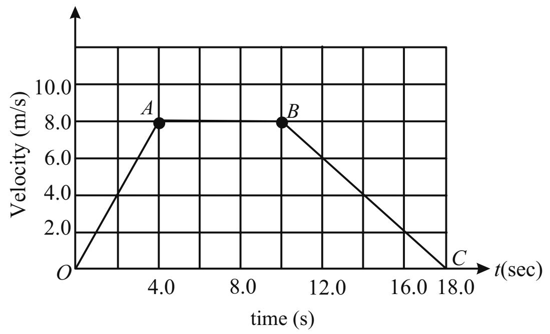
(*a*) 2 m/s^2 (*b*) 4 m/s^2
(*c*) 6 m/s^2 (*d*) 8 m/s^2

Sol. Given $S = 2t^2 + 2t + 4$

$$\therefore \text{ Velocity } (v) = \frac{dS}{dt} = 4t + 2$$

$$\text{Acceleration } (a) = \frac{dv}{dt} = 4(1) + 0 = 4\text{m/s}^2$$

Example 5: What is the acceleration for each graph segment in figure? Describe the motion of the object over the total time interval. Also calculate displacement.



Sol. Segment OA; $a = \dfrac{8-0}{4-0}$ = 2 m/s^2

Segment AB; graph horizontal i.e., slope zero i.e., $a = 0$

Segment BC; $a = \dfrac{0-8}{18-10}$ = – 1 m/s^2

The graph is trapezium. Its area between t = 0 to t = 18s is displacement.

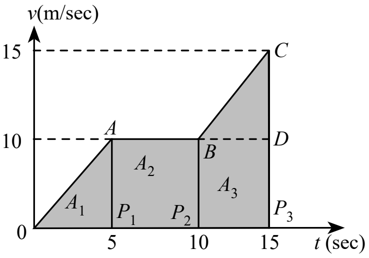
Area of v-t graph = displacement = $\dfrac{1}{2}$ (18 + 6) × 8 = 96 m

Particle accelerates uniformly for first 4 sec., then moves with uniform velocity for next 6 sec. and then retards uniformly to come to rest in next 8 sec.

Example 6: The motion of a body is described in (v–t) graph as given under.

Find the following:

(*a*) Max and Min acceleration
(*b*) Displacement from t = 10 to t = 15
(*c*) Average velocity for the whole journey.



Sol. (*a*) We know slope of (v – t) graph gives acceleration

$$\text{Slope}_{OA} = \frac{AP_1}{OP_1} = \frac{10}{5} = 2\,\text{m/sec}^2 \text{ (Max-acceleration)}$$

$$\text{Slope}_{AB} = 0 \text{ m/sec}^2 \qquad \text{(min} - \text{acceleration)}$$

$$\text{Slope}_{BC} = \frac{CD}{BD} = \frac{5}{5}$$

$$= 1\text{m/sec}^2$$

(*b*) Displacement = Area (v - t)graph from t = 10 to t = 15 sec

$$= \frac{1}{2}(10+15)\times 5 = 62.5 \text{ m}$$

(*c*) $$\text{Average Velocity} = \frac{\text{Total Displacement}}{\text{Total Time}} = \frac{\text{Area } (v \text{ - } t)\text{graph}}{t_{\text{total}}}$$

$$= \frac{A_1 + A_2 + A_3}{t_{total}} = \frac{25+50+62.5}{15}$$

$$= \frac{137.5}{15} = 9.17 \text{ m/sec}$$

Example 7: How long does it take for a particle to travel 100 m if it begins from rest and accelerates at 10 m/s^2? What is its velocity when it has travelled 100 m? What is the average velocity during this time?

Sol. $u = 0$, $a = 10$ m/s^2, $S = 100$ m

Applying $S = ut + \dfrac{1}{2}at^2$

we get $100 = \dfrac{1}{2} \times 10 \times t^2$

$\Rightarrow t = \sqrt{20} = 2\sqrt{5}$ s

$v = u + at = 0 + 10 \times 2\sqrt{5} = 20\sqrt{5}$ m/s

$$v_{\text{avg}} = \frac{u+v}{2} = \frac{0+20\sqrt{5}}{2} = 10\sqrt{5} \text{ m/s}$$

Example 8: A car travelling with 72 km/hr is 30 m from a barrier when the driver slams the breaks. The car hits barrier 2.0 seconds later.

(*a*) What is the car's constant deceleration before impact?
(*b*) How fast is car travelling at impact?

Sol. (*a*) u = 72 km/hr = 72 × $\dfrac{5}{18}$ m/s = 20 m/s

$S = 30$ m
$t = 2$s
$a = ?$

$$S = ut + \frac{1}{2}at^2$$

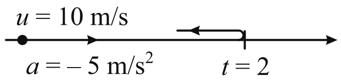
$$30 = 20 \times 2 + \frac{1}{2} \times a \times 2^2$$

$$\Rightarrow a = -\frac{10}{2} = -5 \text{ m/s}^2$$

(*b*) $v = u + at = 20 + (-5) \times 2 = 10$ m/s

Example 9: A particle moving with initial velocity of 10 m/s towards East has an acceleration of 5 m/s^2 towards west. Find the displacement and distance travelled by the particle in first 4 seconds?

Sol.



$v = u + at \Rightarrow 0 = 10 - 5t \Rightarrow t = 2s$

The direction of velocity changes after two seconds.

$$S = 10 \times 4 + \frac{1}{2}(-5) \times 4^2 = 0 = \text{displacement}$$

Distance travelled is not equal to displacement because during course of journey, velocity changes direction.

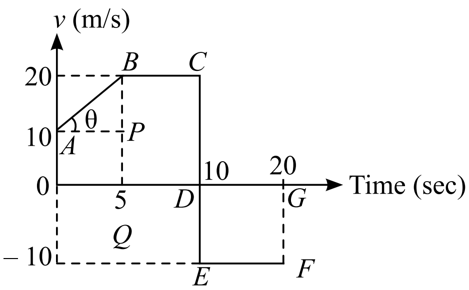
$D = S(\text{at } 2s) + |S(\text{at } 4s) - S(\text{at } 2s)|$

$$= \left(10 \times 2 - \frac{1}{2} \times 5 \times 2^2\right) + \left|0 - (10 \times 2) - \frac{1}{2} \times 5 \times 2^2\right|$$

$= 10 + 10 = 20$ m

Concept Application

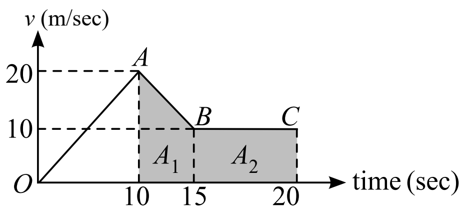
3.



(*a*) Find acceleration between $t = 0$ to $t = 5$

(*b*) Find total displacement between $t = 0$ to $t = 20$ sec.

4.



(*a*) Find the ratio of acceleration for *OA* and *AB* in the graph shown.

(*b*) Total distance covered between 10 to 20 sec.

VERTICAL MOTION UNDER GRAVITY (FREE FALL)

Motion that occurs solely under the influence of gravity is called free fall. Thus a body projected upward or downward or released from rest are all under free fall.

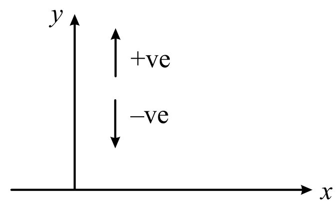
In the absence of air resistance all falling bodies have the same acceleration due to gravity, regardless of their sizes or shapes.

The value of the acceleration due to gravity depends on both latitude and altitude. It is approximately 9.8 m/s^2 near the surface of the earth. For simplicity a value of 10 m/s^2 is used. To do calculations regarding motion under gravity, we follow a proper sign convention. If we take upward direction as positive then $a = -g$

Thus the equation of kinematics may be modified as

$$v = u - gt \quad \text{...(i)}$$

$$\Delta y = y - y_0 = ut - \frac{1}{2}gt^2 \quad \text{...(ii)}$$



$$v^2 = u^2 - 2g(y - y_0) \quad \text{...(iii)}$$

y_0 = position of particle at time $t = 0$

y = position of particle at time t.

u = velocity of particle at time $t = 0$

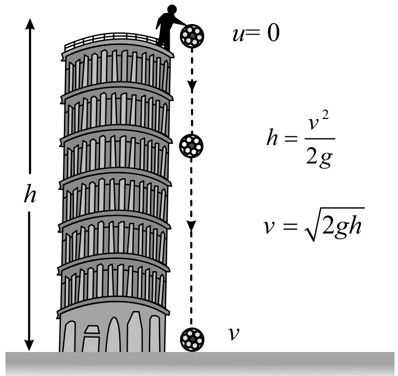
v = velocity of particle at time t.

(*i*) **A body dropped from some height (initial velocity zero)**

- Equation of motion: Taking initial position as origin and downward direction as negative. Here we have,

  $u = 0$ [As body starts from rest]

  $a = -g$ as acceleration is in the downward direction



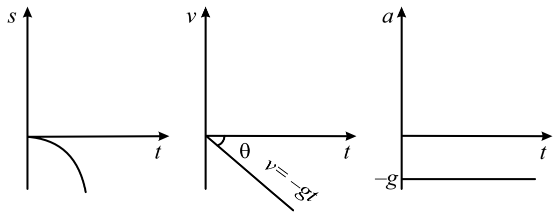
$$v = -gt \quad \text{...(i)}$$

$$\Delta y = -h = -\frac{1}{2}gt^2 \quad \text{...(ii)}$$

$$v^2 = 2(-g)(-h) = 2gh \quad \text{...(iii)}$$

$$h_n = \frac{g}{2}(2n - 1) = \text{Height covered in n}^{\text{th}} \text{ second.} \quad \text{...(iv)}$$

❖ Graph of displacement, velocity and acceleration with respect to time:



❖ As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, $2t$, $3t$, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of integers.

❖ The distance covered in the nth sec, $h_n = \frac{1}{2} g(2n-1)$

So distance covered in 1st sec, 2nd sec, 3rd sec .... etc., will be in the ratio of 1 : 3 : 5. This is called 'Galileo's Law of odd numbers'.

(*ii*) **A body projected vertically downward with some initial velocity:** The initial velocity is downward and will be negative.

Equation of motion: $v = -u - gt$

$\Delta y = -h = -ut - \frac{1}{2} gt^2$

$v^2 = u^2 + 2gh$

$h_n = u + \frac{g}{2}(2n - 1)$

(*iii*) **A body is projected vertically upwards:**

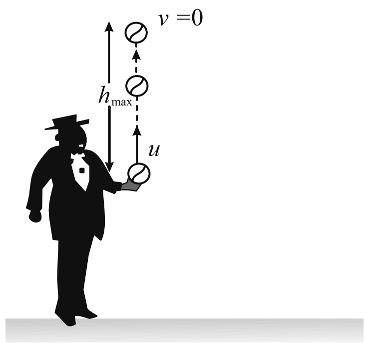
Equation of motion: Taking initial position as origin and vertically up as positive,

$a = -g$ [As acceleration is downwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$v = u - gt$; $h = ut - \frac{1}{2} g t^2$; $v^2 = u^2 - 2gh$;

$h_n = u - \frac{g}{2}(2n - 1)$



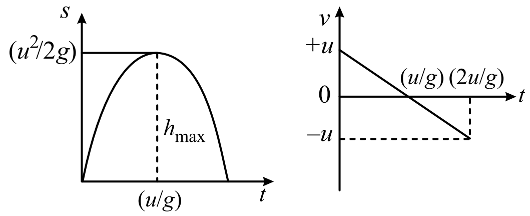
For maximum height $v = 0$

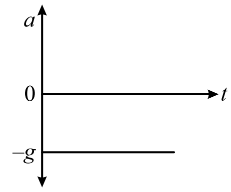
So from above equation

$u = gt \Rightarrow t = \frac{u}{g}$

$h_{max} = \frac{u^2}{g} - \frac{u^2}{2g} = \frac{u^2}{2g}$ Now, $h_{max} = \frac{u^2}{2g} \Rightarrow u = \sqrt{2gh_{max}}$

❖ Graph of displacement, velocity and acceleration with respect to time (for maximum height):





Important Points

(*i*) In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remain constant while speed, velocity, momentum, kinetic energy and potential energy changes.

(*ii*) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$

(*iii*) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance. Time of ascent (t_1) = time of descent (t_2) = u/g

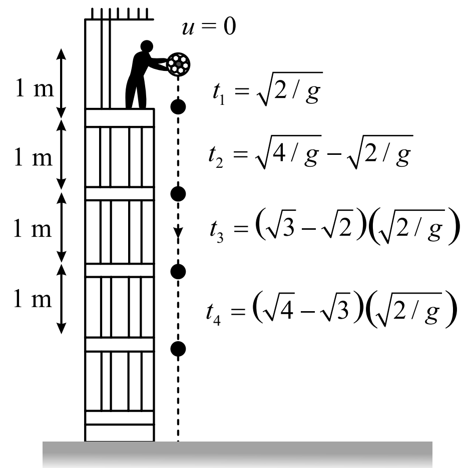
Total time of flight $T = t_1 + t_2 = \frac{2u}{g}$

(*iv*) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

Acceleration at any point on the path is same whether the body is moving in upward or downward direction.

(*v*) A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1 m each will then be in the ratio of the difference in the square roots of the integers i.e.

$\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2}), (\sqrt{4} - \sqrt{3}), \ldots\ldots$



Note:

(*i*) During ascent, $a = -g$, velocity becomes less positive i.e., speed decreases since velocity and acceleration are in opposite direction.

(*ii*) During descent, $a = -g$, but now it is in the direction of velocity so it is not retardation. It makes velocity more negative i.e. speed increases in negative direction.

Train Your Brain

Example 10: A man standing on the top of a building, throws a ball with speed 5 m/s in upward direction from 30 m height above the ground level. How much time does it takes to reach the ground?

Sol. $u = 5$ m/s

When it reaches the ground, $\Delta y = -30$ m

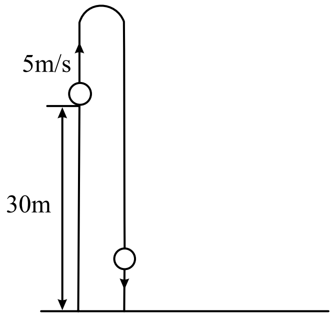
$\therefore$ From above and equation (ii) of motion

$$S = ut - \frac{1}{2}gt^2$$

$$-30 = 5t - \frac{1}{2}(10)t^2$$

$\Rightarrow t^2 - t - 6 = 0$

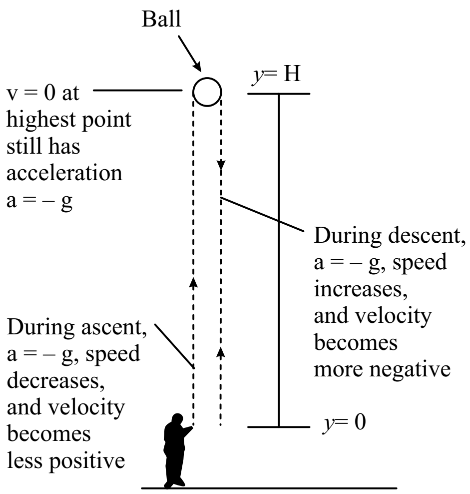
On solving, we get $t = 3$ and -2



Rejecting $t = -2$ sec, we get $t = 3$ sec

Example 11: A kid throws a ball up, with some initial speed. Comment on magnitudes and signs of acceleration and velocity of the ball.

Sol.



Example 12: If a body is thrown up with the velocity of 15 m/s then maximum height attained by the body is ($g = 10$ m/s^2)

(*a*) 11.25 m (*b*) 16.2 m
(*c*) 24.5 m (*d*) 7.62 m

Sol. $H_{max} = \frac{u^2}{2g} = \frac{(15)^2}{2 \times 10} = 11.25\,\text{m}$

Example 13: A body falls from rest in the gravitational field of the earth. The distance travelled in the fifth second of its motion is ($g = 10$ m/s^2)

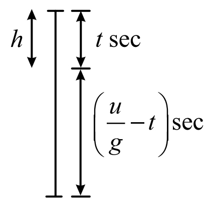
(*a*) 25 m (*b*) 45 m
(*c*) 90 m (*d*) 125 m

Sol. $h_n = \frac{g}{2}(2n-1) \Rightarrow h_{5\text{th}} = \frac{10}{2}(2 \times 5 - 1) = 45$ m

Example 14: If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

(*a*) $\frac{1}{2}gt^2$ (*b*) $ut - \frac{1}{2}gt^2$
(*c*) $(u - gt)t$ (*d*) ut

Sol. If ball is thrown with velocity u, then time of ascent $= \frac{u}{g}$



Velocity after $\left(\frac{u}{g} - t\right)$sec, $v = u - g\left(\frac{u}{g} - t\right) = gt.$

So, distance in last 't' sec, $0^2 = (gt)^2 - 2(g)h.$

$$h = \frac{1}{2}gt^2.$$

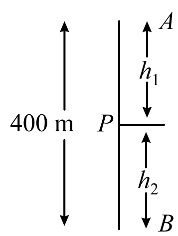
Example 15: A man drops a ball downside from the roof of a tower of height 400 m. At the same time another ball is thrown upside with a velocity 50 m/s from the foot of tower. What is the height from the foot of the tower where the two balls would meet?

(*a*) 100 meters (*b*) 320 meters
(*c*) 80 meters (*d*) 240 meters

Sol. Let both balls meet at point P after time t.

The distance travelled by ball A

$$h_1 = \frac{1}{2}gt^2 \qquad \text{...(i)}$$



The distance travelled by ball B

$$h_2 = ut - \frac{1}{2}gt^2 \quad \text{...(ii)}$$

By adding (i) and (ii) $h_1 + h_2 = ut = 400$

(Given $h = h_1 + h_2 = 400$)

$\therefore$ $t = 400/50 = 8$ s and $h_1 = 320$ m, $h_2 = 80$ m

Example 16: Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant?

(*a*) 2.50 m (*b*) 3.75 m
(*c*) 4.00 m (*d*) 1.25 m

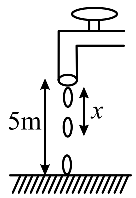
Sol. Let the interval between each drop be t then from question

For first drop $\frac{1}{2}g(2t)^2 = 5$ …(i)

For second drop $x = \frac{1}{2}gt^2$ …(ii)

By solving (i) and (ii) $x = \frac{5}{4}$ and

Hence required height $h = 5 - \frac{5}{4} = 3.75m.$



Example 17: A balloon is at a height of 81 m and is ascending vertically upward with a velocity of 12 m/sec. A body of 2 kg weight is dropped from it. If $g = 10$ m/s^2 the body will reach the surface of the earth in

(*a*) 1.5 s (*b*) 4.025 s
(*c*) 5.4 s (*d*) 6.75 s

Sol. As the balloon is going up so initial velocity of balloon = + 12 m/s,

$\Delta y = -81$ m; $a = -g = -10$ m/s^2

By applying $h = ut + \frac{1}{2}gt^2$; $-81 = 12t - \frac{1}{2}(10)t^2$

$\Rightarrow 5t^2 - 12t - 81 = 0$

$\Rightarrow t = \dfrac{12 \pm \sqrt{144 + 1620}}{10} = \dfrac{12 \pm \sqrt{1764}}{10}$

= 5.4 s

Example 18: A particle is dropped under gravity from rest from a height h ($g = 9.8$ m/s^2) and it travels a distance $\frac{9h}{25}$ in the last second, the height h is

(*a*) 100 m (*b*) 122.5 m
(*c*) 145 m (*d*) 167.5 m

Sol. Distance travelled in n sec $= \frac{1}{2}gn^2 = h$ …(i)

Distance travelled in n^{th} sec.

$= \frac{g}{2}(2n-1) = \frac{9h}{25}$ …(ii)

Solving (i) and (ii)

We get. $h = 122.5$ m

Example 19: A stone is thrown vertically upward with a speed u from the top of the tower, reaches the ground with a velocity $3u$. The height of the tower is

(*a*) $3u^2/g$ (*b*) $4u^2/g$ (*c*) $6u^2/g$ (*d*) $9u^2/g$

Sol. Initial velocity = $-u$.

By applying $v^2 = u^2 + 2gh$,

$(3u)^2 = (-u)^2 + 2gh$,

$\Rightarrow h = \dfrac{4u^2}{g}$

Example 20: A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is

(*a*) 4.9 m (*b*) 9.8 m
(*c*) 19.6 m (*d*) 24.5 m

Sol. The separation between two bodies, two seconds after the release of second body is given by

$$s = \frac{1}{2}g(t_1^2 - t_2^2) = \frac{1}{2} \times 9.8 \times (3^2 - 2^2) = 24.5\ m$$

Concept Application

5. A body is projected from top of a tower vertically upward with 5 m/sec, the body lands the ground in 4 sec. Total height of tower would be

(*a*) 45 m (*b*) 60 m (*c*) 30 m (*d*) 50 m

6. A body is thrown vertically upwards and if returns back to hand in 3 sec, its velocity of throw and max-height attained are,

(*a*) 15 m/sec, 11.25 m (*b*) 10 m/sec, 12 m
(*c*) 30 m/sec, 22.5 m (*d*) 20 m/sec, 30 m

7. Water drops are released from the bottom of an overhead tank at regular interval. When 1st drop touches the ground, fifth drop is just about to release. The height of overhead tank is 8m from the ground. Find the separation between 2nd and 3rd drop just when 1st drop touches the ground.

(*a*) 1.5 m (*b*) 2 m
(*c*) 2.5 m (*d*) 4.5 m

8. Two balls A & B of masses m & 5 m are dropped from the towers of height 36 m and 64 m respectively. The ratio of time taken by them to reach the ground.
   (*a*) 2/3 (*b*) 7/4 (*c*) 5/2 (*d*) 3/4
9. A splash is heard 3 sec after the stone is released into a well of depth 20 m. The velocity of sound is
   (*a*) 20 m/sec (*b*) 40 m/sec
   (*c*) 10 m/sec (*d*) 35 m/sec

MOTION WITH VARIABLE ACCELERATION

In previous section, we studied rectilinear motion when acceleration is constant. In general acceleration can vary and depend on time, position and velocity of the particle.

Let us consider some simple cases.

(*i*) **Acceleration only depends on time *t*.**

$$\frac{dv}{dt} = a(t)$$

$$\int_u^v dv = \int_0^t a(t)dt \Rightarrow v - u = \int_0^t a(t)dt$$

or $v = u + \int_0^t a(t)dt$

(*ii*) **Acceleration only depends on position *x*.**

$$\frac{dv}{dt} = a(x)$$

We can use chain rule to eliminate time.

$$\frac{dv}{dx}\frac{dx}{dt} = a(x) \Rightarrow \text{v}\frac{dv}{dx} = a(x)$$

$$\int_u^v vdv = \int_{x_0}^x a(x)dx$$

or $\frac{v^2}{2} - \frac{u^2}{2} = \int_{x_0}^x a(x)dx$

(*iii*) **Acceleration only depends on velocity.**

$$\frac{dv}{dt} = a(v) \Rightarrow \int_u^v \frac{dv}{a(v)} = \int_0^t dt$$

or $\int_u^v \frac{dv}{a(v)} = t$

This gives us velocity as a function of time.

In case we want velocity as a function of position, we can use chain rule.

$$\frac{dv}{dx}\frac{dx}{dt} = a(v) \Rightarrow \frac{v\,dv}{a(v)} = dx$$

$$\int_u^v \frac{v\,dv}{a(v)} = \int_{x_0}^x dx = x - x_0$$

Train Your Brain

Example 21: The acceleration a of a particle moving in one dimension is given by $a = 6 - 2t$. If the particle is initially at $x = 0$ and its velocity is 2 m/s, find its position and velocity at time t.

Sol. $\frac{dv}{dt} = 6 - 2t$

$$\int_2^v dv = \int_0^t (6 - 2t)dt$$

$\Rightarrow v - 2 = (6t - t^2)|_0^t = 6t - t^2 \Rightarrow v(t) = 2 + 6t - t^2$

To find position, we integrate velocity.

$v = \frac{dx}{dt} = 2 + 6t - t^2 \Rightarrow dx = (2 + 6t - t^2)dt$

$$\int_0^x dx = \int_0^t (2 + 6t - t^2)dt = 2t + 3t^2 - \frac{t^3}{3}$$

or $x(t) = 2t + 3t^2 - \frac{t^3}{3}$

Example 22: The retardation of a car when its engine is shut off depends on its velocity as $a = -\alpha v$ where α is positive constant. Find the total distance travelled by the car if its initial velocity is 20 m/s and $\alpha = 0.5$/s.

Sol. $\frac{dv}{dt} = -\alpha v$

$$\frac{dv}{dx}\left(\frac{dx}{dt}\right) = -\alpha v \Rightarrow \frac{vdv}{dx} = -\alpha v$$

or $dv = -\alpha dx$

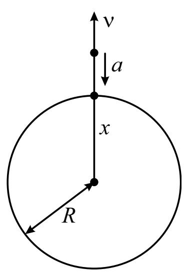
$$\int_{20}^0 dv = -\alpha\int_0^d dx \Rightarrow v|_{20}^0 = -\alpha x|_0^d$$

$-20 = -\alpha d$

$$d = \frac{20}{\alpha} = \frac{20}{0.5} = 40 \text{ m}$$

Example 23: With what velocity in vertical upward direction should a body be projected from the surface of earth so that it reaches a height equal to radius of earth? The acceleration of body is given by $a = -\frac{GM}{x^2}$ where x is the distance from centre of earth and M is the mass of earth.

Sol. Note that acceleration due to gravity is nearly constant near the surface of earth. But if the height become too large its dependence on distance can not be ignored.



$$a = \frac{dv}{dt} = -\frac{GM}{x^2}$$

or $\frac{dv}{dx}.\frac{dx}{dt} = -\frac{GM}{x^2} \Rightarrow vdv = = -\frac{GM}{x^2}dx$

At the highest point, velocity is zero. Also note $x_i = R$ and $x_f = 2R$.

$$\int_u^0 vdv = -GM\int_R^{2R}\frac{dx}{x^2}$$

$$\left.\frac{v^2}{2}\right|_u^0 = -GM\int_R^{2R}x^{-2}dx = \left.\frac{GM}{x}\right|_R^{2R}$$

$$\Rightarrow -\frac{u^2}{2} = GM\left[\frac{1}{2R} - \frac{1}{R}\right]$$

$$\Rightarrow u^2 = \frac{GM}{R} \Rightarrow u = \sqrt{\frac{GM}{R}} = \sqrt{\frac{GM}{R^2}R}$$

$$\because g = \left(\frac{GM}{R^2}\right)$$

$$= \sqrt{gR} = 8 \text{ km/s} \quad [\because R = 6400 \text{ km, g} = 10 \text{ m/s}^2]$$

Concept Application

10. Starting from rest at $t = 0$, a particle moves in a straight line with an acceleration a given by $a = t^3$ m/s^2 where t is in seconds. Then the velocity of particle after 4 seconds is

(*a*) 32 m/s (*b*) 64 m/s
(*c*) 128 m/s (*d*) 16 m/s

11. A particle moves in a straight line with acceleration $a = -\frac{1}{3v^2}$ where v is its velocity at time t. If initial velocity is 5 m/s then time t at which its velocity becomes zero is

(*a*) 5 sec (*b*) 25 sec (*c*) 125 sec (*d*) 50 sec

12. The acceleration of a particle as a function of its position x is given $a = -2x$. If velocity at $x = 0$ is 20 m/s, find the position x where its velocity becomes zero.

(*a*) $10\sqrt{2}$ m (*b*) $5\sqrt{2}$ m
(*c*) $20\sqrt{2}$ m (*d*) 20 m

RELATIVE MOTION

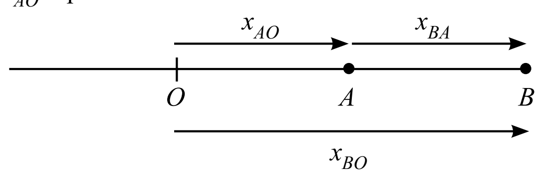
Every motion is actually only relative motion. There is nothing like absolute motion. Whether a body is moving or is at rest is not a quality of body itself rather it is always with respect to an observer. If the observer finds that the position of an object is not changing when observed by him then the object is 'actually' at rest (and vice-versa). Therefore, when a passenger (A) in a train observes another passenger (B) then he finds that passenger (and the train) to be not moving. Therefore, it means that the passenger and train both are in reality at rest for passenger (A). However, another observer on platform finds the train and passenger (B) to be moving when they are in reality moving for this observer only. Due to this duality we say that motion is not absolute rather it is dependent on observer.

Relative position of B with respect to A,

$$x_{BA} = x_{BO} - x_{AO} \quad \text{...(i)}$$

where, x_{BO} = position of B wrt. O

x_{AO} = position of A wrt O



Differentiating eq. (i) gives us the relation for relative velocity

$$\frac{d}{dt}x_{BA} = \frac{d}{dt}x_{BO} - \frac{d}{dt}x_{AO}$$

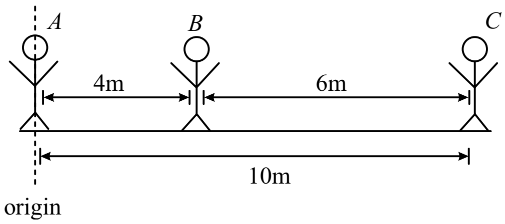
$$V_{BA} = V_{BO} - V_{AO}$$

Furthur differentiating,

$$a_{BA} = a_{BO} - a_{AO}$$

Train Your Brain

Example 24: The position of three men A, B and C is shown in figure. Then find position of one man with respect to other (take +ve direction towards right and –ve towards left)



Sol. Here,

Position of B w.r.t. A is 4 m towards right. ($x_{BA} = +4$ m)

Position of C w.r.t. A is 10 m towards right. ($x_{CA} = +10$m)

Position of C w.r.t. B is 6 m towards right ($x_{CB} = +6$m)

Position of A w.r.t. B is 4 m towards left. ($x_{AB} = -4$m)

Position of A w.r.t. C is 10 m towards left. ($x_{AC} = -10$m)

Example 25: Two objects A and B are moving towards each other with velocities 10 m/s and 12 m/s respectively as shown.

(*i*) Find the velocity of A with respect to B.

(*ii*) Find the velocity of B with respect to A

Sol. $v_A = +10$ m/s, $v_B = -12$ m/s

(*i*) $v_{AB} = v_A - v_B = (10) - (-12) = 22$ m/s.

(*ii*) $v_{BA} = v_B - v_A = (-12) - (10) = -22$ m/s.

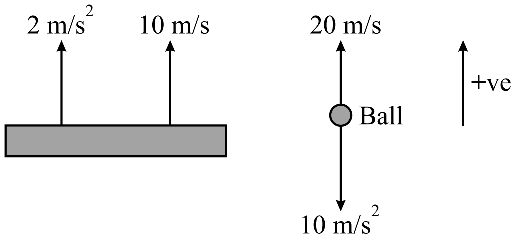
Example 26: An open lift is moving upwards with velocity l0m/s. It has an upward acceleration of 2 m/s^2. A ball is projected upwards with velocity 20 m/s relative to ground. Find :

(*a*) Time when ball again meets the lift.

(*b*) Displacement of lift and ball at that instant.

(*c*) Distance travelled by the ball upto that instant. Take $g = 10$ m/s^2

Sol. (*a*) At the time when ball again meets the lift,



$L \rightarrow$ Lift $\qquad B \rightarrow$ Ball

$\therefore\ 10t + \frac{1}{2} \times 2 \times t^2 = 20\,t - \frac{1}{2} \times 10t^2$

Solving this equation, we get

$t = 0$ and $t = \frac{5}{3}$ s

$\therefore$ Ball will again meet the lift after $\frac{5}{3}$ s.

(*b*) At this instant

$$S_L = S_B = 10 \times \frac{5}{3} + \frac{1}{2} \times 2 \times \left(\frac{5}{3}\right)^2 = \frac{175}{9} = 19.4\text{m}$$

(*c*) For the ball $u\uparrow\ \downarrow a$. Therefore, we will first find t_0, the time when its velocity becomes zero.

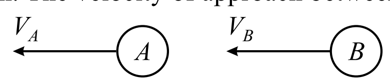
As $t\left(=\frac{5}{3}s\right) < t_0$ distance and displacement are equal

or $d = 19.4$ m

Concept of relative motion is more useful in two body problem in two (or three) dimensional motion. This can be understood by the following example.

Concept Application

13. Two particles are moving along a straight line as shown. The velocity of approach between *A* and *B* is



(*a*) $V_A + V_B$ (*b*) $|V_A - V_B|$

(*c*) $V_A - V_B$ (*d*) $V_B - V_A$

AARAMBH (SOLVED EXAMPLES)

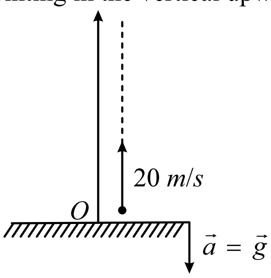
1. A stone is thrown vertical upwards from ground level with $u = 20$ m/s.

(*a*) Find the maximum height attained by the stone.

(*b*) time interval t after which it returns to the point of projection.

(*c*) The velocity with which it strikes the ground.

Sol. Let us choose our origin O at the point of projection with +ve X-axis pointing in the vertical upwards direction.



Note that in this coordinate system, acceleration due to gravity is negative because it points in the downward direction.

Thus $a = -9.8$ m/s^2, $u = 20$ m/s

(*a*) At the highest point, velocity of the particle will become zero. Let h be the maximum height.

Thus $S = h$.

Using the relation,

$v^2 - u^2 = 2a\,S$

we get $0 - 20^2 = 2 \times (-9.8) \times h$

$$\Rightarrow h = \frac{400}{19.6} = 20.4\,\text{m}$$

Therefore, 20.4 m is the correct answer.

(*b*) $S = 0 = 20t - \frac{1}{2}(9.8)t^2$

$$\Rightarrow t = \frac{40}{9.8} = 4.08 \text{ sec}$$

Therefore, 4.08 s is the correct answer.

(*c*) Since, the particle returns to the initial position, $S = 0$.

$$\Rightarrow v = 20 - 9.8 \times \frac{40}{9.8} \quad \text{(we know } t \text{ from part (}b\text{))}$$

$= -20$ m/s

Here, minus sign indicates that particle moves in the downward direction.

Therefore, –20 m/s is the correct answer.

Note: It returns with same speed with which it was thrown.

2. A body is thrown down from the top of a tower of height h with velocity 10 m/s. Simultaneously, another body is projected upward from bottom. They meet at a height $2h/3$ from the ground level. If $h = 60$ m, find the initial velocity of the lower body.

(*a*) 19.23 m/s (*b*) 38.27 m/s

(*c*) 55.16 m/s (*d*) None of these

Sol. Let us choose the origin at the ground level with +ve y-axis pointing in the upward direction.

Let us refer lower and upper body as 1 and 2 respectively.

Then,

$a = -g$

$x_{1_i} = 0$, $x_{1_f} = 2h/3$, $u_1 = ?$

$x_{2_i} = h$, $x_{2_f} = 2h/3$, $u_2 = -10$ m/s

From eqn of motion , we have

$$x_{1_f} = 0 + u_1 t - \frac{1}{2} \times 9.8 \times t^2 \quad \text{...(i)}$$

$$x_{2_f} = h - 10t - \frac{1}{2} \times 9.8 \times t^2 \quad \text{...(ii)}$$

But , $x_{1_f} = x_{2_f} = 2h/3$

Hence, equating eqn (i) and (ii) we have

$$u_1 t = h - 10\,t \quad \Rightarrow t = \frac{h}{u_1 + 10}$$

Putting this value in eqn (ii) , we get

$$\frac{h}{3} = \frac{10h}{u_1 + 10} + 4.9\left(\frac{h}{u_1 + 10}\right)^2$$

But, $h = 60$ m.

$$\Rightarrow 20 = \frac{600}{u_1 + 10} + 4.9\frac{60^2}{(u_1 + 10)^2}$$

$$\Rightarrow (u_1 + 10)^2 - 30(u_1 + 10) - 882 = 0$$

Solving this quadratic eqn, we find

$$u_1 + 10 = \frac{30 \pm \sqrt{100 + 3528}}{2} \qquad \Rightarrow u_1 = 38.27 \text{ m/s}$$

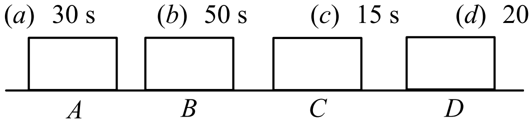
The other value is not possible because body is thrown upwards and is positive in the chosen coordinate system.

Therefore, option (*b*) is the correct answer.

3. A car starts moving on a straight road, first with acceleration $a = 5$ m/s^2, then moves uniformly, and finally decelerating at the same rate, comes to rest. The total time of motion equals 25 sec. The average velocity during that time is 72 km/hr. How long did the car move uniformly?

(*a*) 30 s (*b*) 50 s (*c*) 15 s (*d*) 20 s

Sol.



Let AB, BC and CD be the displacements of the car when it accelerates, moves with constant velocity and decelerates respectively.

$$< v >= 72 \text{ km/hr} = 72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$$

Total distance travelled = $< v > \times$ time = $20 \times 25 = 500$ m.

From $A \rightarrow B$

$$AB = ut_{AB} + \frac{1}{2}a(t_{AB})^2$$

$= 2.5(t_{AB})^2$

$v_B = 0 + at_{AB} = 5t_{AB}$

From $B \to C$

Since velocity is constant

$BC = v_B \times t_{BC} = 5t_{AB} \times t_{BC}$

From $C \to D$

In this interval

$u = v_B = 5t_{AB},\ v = 0,\ a = -5$ m/s^2

$v = u + at \Rightarrow 0 = 5t_{AB} - 5t_{CD} = 0$

$\Rightarrow t_{AB} = t_{CD}$ and

$S = ut - \frac{1}{2}at^2$

$\Rightarrow CD = 5t_{AB}^2 - 2.5\left(t_{AB}\right)^2 \quad = 2.5t_{AB}^2 = AB$

Total time $= t_{AB} + t_{BC} + t_{CD} = 25$

$\Rightarrow 2t_{AB} + t_{BC} = 25$ ...(i)

Also,

Total distance $= AB + BC + CD = 500$

$\Rightarrow 2.5t_{AB}^2 + 5t_{AB}\ t_{BC} + 2.5t_{AB}^2 = 500$ ...(ii)

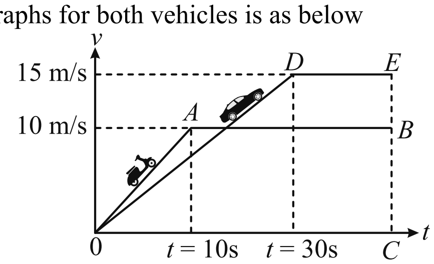
Solving eqn (i) and eqn (ii) we find $t_{BC} = 15$ sec

Therefore, option (*c*) is the correct answer.

4. A motorcycle and a car start their rectilinear motion from rest from the same place at the same time and travel in the same direction. The motorcycle accelerates at 1.0 m/s^2 up to a speed of 36 km/hr and the car at 0.5 m/s^2 up to a speed of 54 km/hr. Their velocities remain constant after that. Draw *v-t* graph of both. Calculate the distance at which the car would overtake the motorcycle.

(*a*) 150 m (*b*) 900 m (*c*) 300 m (*d*) 100 m

Sol. *v-t* graphs for both vehicles is as below



For motorcycle

10 m/s = 0 + (1m/s^2) t

t = 10s

For car

15 m/s = 0 + (0.5 m/s^2) t

t = 30 s

Suppose after t time car over takes motorcycle.

Area under *v-t* graph till that time for both will be same

Area of *OABC* = Area of *ODEC*

$\frac{1}{2}(t + t - 10)\ 10 = \frac{1}{2}(t + t - 30)\ 15$

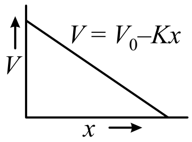
$4t - 20 = 6t - 90$

$2t = 70 \Rightarrow t = 35$ sec.

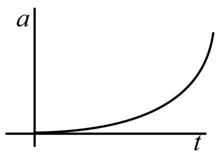
Area under the graph = 300 m = distance at which car overtakes motorcycle.

Therefore, option (*c*) is the correct answer.

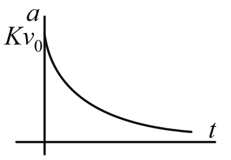
5. A particle is moving along *x*-axis with velocity *V* which varies according to the law $V = V_0 - Kx$ here V_0 and K are constants. Choose the correct acceleration *vs* time plot for the time interval when particle moves from $x = 0$ to $x = \frac{V_0}{K}$.



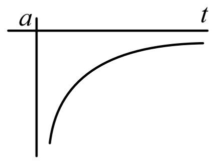
(*a*)



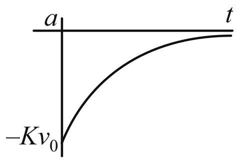
(*b*)



(*c*)



(*d*)



Sol. $V = V_0 - Kx$

$\frac{dx}{dt} = (V_0 - Kx) \Rightarrow \int_0^x \frac{dx}{(V_0 - Kx)} = \int_0^t dt$

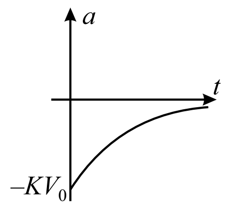
$x = \frac{V_0}{K}(1 - e^{-Kt})$

$\therefore \frac{dx}{dt} = +V_0\, e^{-Kt} \Rightarrow a = \frac{d^2x}{dt^2} = -KV_0\, e^{-Kt}$

At $t = 0$, $a = -KV_0$

At $t = \infty$, $a = 0$

Therefore, graph is as shown



Therefore, option (*d*) is the correct answer.

6. A steel ball bearing is released from the roof of a building. An observer standing in front of a window 120 cm high observes that the ball takes 0.125 sec to fall from top to the bottom of the window. The ball continues to fall and makes a completely elastic collision with side walk and reappears at the bottom of the window 2 s after passing it on the way down. How tall is the building? (In elastic collision the kinetic energy remains conserved before and after collision)

(*a*) 50 m (*b*) 41.5 m

(*c*) 75 m (*d*) 20.5 m

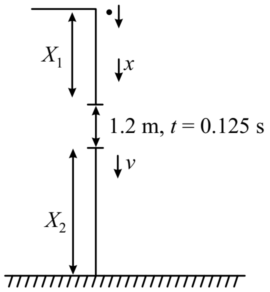
Sol. $1.2 = ut + \frac{1}{2}gt^2$

$1.2 = u \times 0.125 + \frac{1}{2} \times 10 \times (0.125)^2$

$1.2 = u \times 0.125 + 5 \times (0.125)^2$

$u = \frac{1.2 - 0.078125}{0.125} = 8.975$ m/s

$v = 8.975 + 10 \times 0.125 = 10.225$ m/s



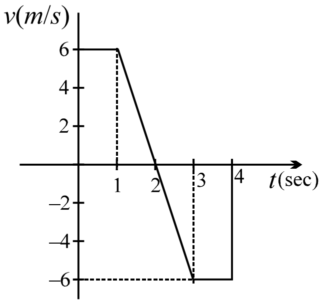
$X_2 = 10.225 \times 1 + \frac{1}{2} \times 10 \times 1^2 = 15.225$ m

$u^2 = 0 + 2 \times 10 \times X_1 \Rightarrow X_1 = \frac{u^2}{20} = \frac{(8.975)^2}{20} = 4.028$ m

$H_{total} = X_1 + X_2 + 1.2 = 4.028 + 15.225 + 1.2$
$= 20.4525 = 20.5$ m

Therefore, option (*d*) is the correct answer.

7. A particle moves along a straight line along *x*-axis. At time $t = 0$, its position is at $x = 0$. The velocity *v* m/s of the object changes as a function of time *t* seconds as shown in the figure.



(*i*) What is *x* at $t = 1$ sec?
(*a*) 2 m (*b*) 4 m (*c*) 6 m (*d*) 8 m

(*ii*) What is the acceleration at $t = 2$ sec?
(*a*) 10 m/s^2 (*b*) 20 m/s^2 (*c*) –12 m/s^2 (*d*) –6 m/s^2

(*iii*) What is *x* at $t = 4$ sec?
(*a*) 0 m (*b*) 1 m (*c*) 5 m (*d*) 10 m

(*iv*) What is the average speed between $t = 0$ and $t = 3$ sec?
(*a*) 8 m/s (*b*) 4 m/s (*c*) 2 m/s (*d*) 1 m/s

Sol. (*i*) *x* is displacement at $t = 1$ sec.
Area under the $v - t$ curve gives displacement
From $t = 0$ to $t = 1$ sec.
$x = 6 \times 1 = 6$ m
Therefore, option (*c*) is the correct answer.

(*ii*) Slope of the *v*–*t* curve gives acceleration from the given *v*–*t* curve
Slope at $t = 2$ sec. gives acceleration at $t = 2$ sec.

$\tan\theta = a = -\frac{6}{1} = -6$ m/s^2

Therefore, option (*d*) is the correct answer.

(*iii*) *x* (at $t = 4$ sec):
Area under the curve from $t = 0$ to $t = 4$ sec

$= 6 \times 1 + \frac{1}{2} \times 6 \times 1 - \frac{1}{2} \times 6 \times 1 - 6 \times 1 = 0$

$\Rightarrow x\,(t = 4) = 0$ m
Therefore, option (*a*) is the correct answer.

(*iv*) Average speed from $t = 0$ to $t = 3$ sec.
Displacement from $t = 0$ to $t = 2$ sec. = Area under the curve $= 6 + \frac{1}{2} \times 6 \times 1 = 9$ m
Displacement form $t = 2$ to $t = 3$ sec. $= -\frac{1}{2} \times 6 \times 1 = -3$ m
Distance from $t = 0$ to $t = 3$ sec $= |9| + |-3| = 12$ m

Average speed $= \frac{\text{Distance}}{\text{Time}} = \frac{12}{3} = 4$ m/s

Therefore, option (*b*) is the correct answer.

8. When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appears on the road, then the time elasped before he slams the breaks of the car is the reaction time. Reaction time depends on complexity of the situation and on individual. You can measure your reaction time by a simple experiment. Take a ruler and ask your friend to drop it vertically through the gap between your thumb and forefinger. After you catch it, find the distance d travelled by the ruler. In a particular case, *d* was found to be 20 cm. [$g = 10$ m/s^2]

(*i*) Estimate reaction time.
(*a*) 0.1 s (*b*) 0.2 s (*c*) 0.4 s (*d*) 1 s

(*ii*) Now if you are driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2, find the distance travelled by the car after you see the need to put the brakes on.
(*a*) 21.75 m (*b*) 42.5 m (*c*) 10.875 m (*d*) 85 m

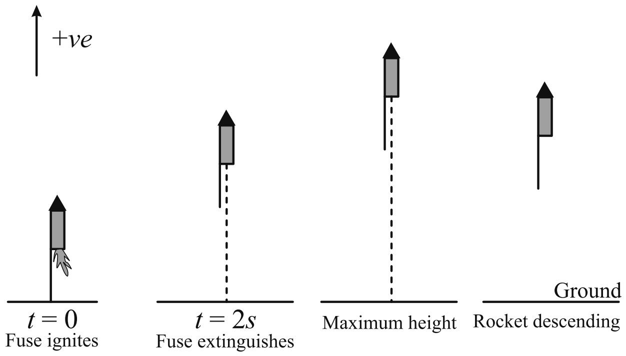
Sol.

(*i*) $0.2 = \frac{1}{2} \times 10t^2 \Rightarrow t^2 = \frac{0.4}{10} \Rightarrow t = 0.2\ s$
Reaction time = 0.2 sec
Therefore, option (*b*) is the correct answer.

(*ii*) 54 km/hr $= \frac{54 \times 5}{18} = 15$ m/s
Total distance = 15 × reaction time + ($v^2/2a$)

Total distance $= 0.2 \times 15 + \frac{15^2}{2 \times 6} = 3 + \frac{225}{12}$
= 21.75 m
Therefore, option (*a*) is the correct answer.

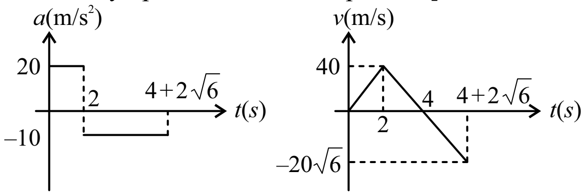
9. A Diwali rocket is launched vertically with its fuse ignited at time $t = 0$, as shown. The charge provides constant acceleration for 2 sec. till rocket attains $V = 40$ ms^{-1}. Afterwards rocket continues to move freely under gravity.



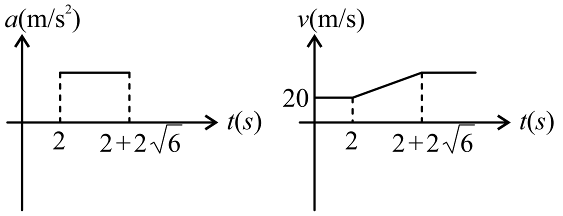
The acceleration-time and velocity-time graphs for the rocket from lauching till it reaches ground are

[Take vertically upward direction as positive]

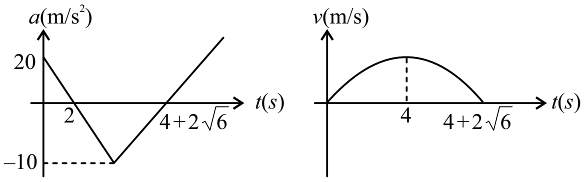
(*a*)



(*b*)



(*c*)



(*d*) None of these

Sol. $v = u + at$

$\Rightarrow 40 = 0 + a \times 2 \Rightarrow a = 20 \text{ m/s}^2$

$v^2 = u^2 + 2ah \Rightarrow (40)^2 = 0 + 2 \times 20h$

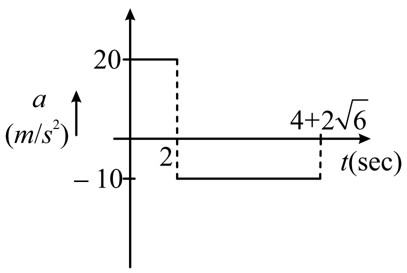
$\Rightarrow h = \dfrac{40 \times 40}{40} = 40 \text{ m}$

Time taken in reaching from $h = 40$ m to ground

$h = ut - \dfrac{1}{2}gt^2 \Rightarrow -40 = 40t - \dfrac{1}{2} \times 10 \times t^2$

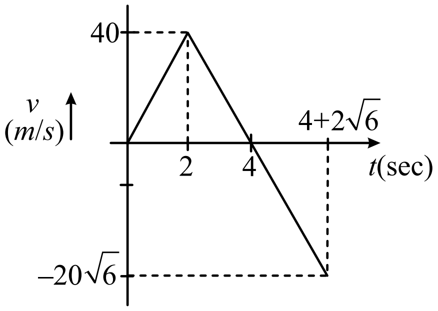
On solving

$t = 4 + 2\sqrt{6}$ sec



$v^2 = 40^2 + 2(-10)(-40)$

$\Rightarrow v = -20\sqrt{6}$ m/s



Therefore, option (*a*) is the correct answer.

10. A block of mass m is fired horizontally along a level surface that is lubricated with oil. The oil provides a viscous resistance that varies as the 3/2 power of the speed. If the initial speed of the block is v_0 at $x = 0$, find the maximum distance reached by the block. Assume no resistance to motion other than that provided by the oil.

(*a*) $mv_0^{1/3}$ (*b*) $2mv_0^{1/3}$

(*c*) $3mv_0^{1/3}$ (*d*) $4mv_0^{1/3}$

Sol. $F = -v^{3/2}$

$a = -\dfrac{1}{m}v^{3/2}$

$v\dfrac{dv}{dx} = -\dfrac{1}{m}v^{3/2}$

$\int_{v_0}^{0} v^{-\frac{1}{2}}dv = -\dfrac{1}{m}\int_0^d dx$

$2mv_0^{1/2} = d$ or $d = 2mv_0^{1/2}$

Therefore, option (*b*) is the correct answer.

11. Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where x is position in metre and a in m/s^2). It is at rest at $x = 0$. At what position x (in metre) will the particle again come to instantaneous rest?

(*a*) 2 m (*b*) 3 m (*c*) 4 m (*d*) 5 m

Sol. $\dfrac{vdv}{dx} = 4 - 2x$

$\int_0^v vdv = \int_0^x (4-2x)dx \Rightarrow \dfrac{v^2}{2} = 4x - x^2$

when $v = 0$, $4x - x^2 = 0$

$x = 0, 4$

$\therefore$ At $x = 4$ m, the particle will again come to rest.

Therefore, option (*c*) is the correct answer.

12. The driver of a train A running at 25 ms^{-1} sights a train B moving in the same direction on the same track with 15 ms^{-1}. The driver of train A applies brakes to produce a deceleration of 1.0 ms^{-2}. To avoid accident the minimum distance between two trains should be

(*a*) 20 m (*b*) 30 m (*c*) 50 m (*d*) 70 m

Sol. $v_r = 25 - 15 = 10$ m/s and $a_r = -1$ m/s^2 so by $v_r^2 = u_r^2 + 2a_rS$

$S = \dfrac{100}{2 \times 1} = 50$ m.

Therefore, option (*c*) is the correct answer.

13. The driver of a car travelling at a speed of 20 m/s, wishes to overtake a truck that is moving with a constant speed of 20 m s^{-1} in the same lane. The car's maximum acceleration is 0.5 m s^{-2}. Initially the vehicles are separated by 40 m, and the car returns back into its lane after it is 40 m ahead of the truck. The car is 3 m long and the truck 17 m. Based on the above comprehension, answer the following questions.

(*i*) Find the minimum time required for the car to pass the truck and return back to its lane?

(*a*) 30 s (*b*) 20 s (*c*) 40 s (*d*) 70 s

(*ii*) What distance does the car travel during this time?
(*a*) 250 m (*b*) 350 m (*c*) 600 m (*d*) 500 m

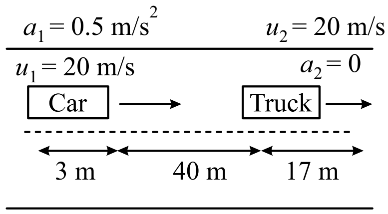
(*iii*) What is the final speed of the car ?
(*a*) 30 ms^{-1} (*b*) 25 ms^{-1} (*c*) 35 ms^{-1} (*d*) 20 ms^{-1}

Sol. (*i*) Displacement of car relative to truck

$x_r = 40 + 17 + 3 + 40$

$= 100$ m.

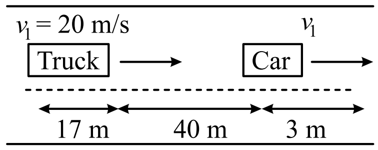
$t = 0$ (Initilly)



Relative initial velocity between car and truck

$u_r = 20 - 20 = 0$

$t = t$ (Finally)



Relative acceleration between car and truck

$a_r = 0.5 - 0 = 0.5$ m/s^2

Let required time = t.

$\therefore$ II equation of motion

$$x_r = u_r t + \frac{1}{2} a_r t^2 \Rightarrow 100 = 0 + \frac{1}{2} \times 0.5 \times t^2$$

$\Rightarrow t = 20$ sec.

Therefore, option (*b*) is the correct answer.

(*ii*) Distance travelled by car

$$x_c = ut + \frac{1}{2}at^2$$

$$= 20 \times 20 + \frac{1}{2} \times 0.5 \times 20^2 = 500 \text{ m}$$

Therefore, option (*d*) is the correct answer.

(*iii*) Final speed of the car

$= u + at$

$= 20 + 0.5 \times 20 = 30$ m/s.

Therefore, option (*a*) is the correct answer.

14. Men are running along a road at 15 km/h behind one another at equal intervals of 20 m. Cyclists are riding in the same direction at 25 km/h at equal intervals of 30 m. What should be the speed of an observer travelling along the road in opposite direction so that whenever he meets a runner he also meets a cyclist? (neglect the size of cycle)

(*a*) 10 kmh^{-1} (*b*) 7 kmh^{-1} (*c*) 15 kmh^{-1} (*d*) 5 kmh^{-1}

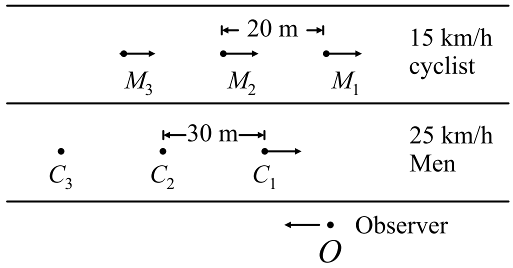
Sol. Let u = speed of observer.

Relative velocity between observer and a man

$= u + 15$ km/h.

Relative velocity between observer and a cyclist.

$= u + 25$ km/h.



Hence, to for a man and a cyclist to meet simultaneously

$$\frac{20 \text{ m}}{(u+15) \text{ km/h}} = \frac{30 \text{ m}}{(u+25) \text{ km/h}}$$

$\Rightarrow u = 5$ km/h

Therefore, option (*d*) is the correct answer.

15. Car A and car B start moving simultaneously in the same direction along the line joining them. Car A moves with a constant acceleration $a = 4$ m/s^2, while car B moves with a constant velocity $v = 1$ m/s. At time $t = 0$, car A is 10 m behind car B. Find the time when car A overtakes car B. How much time A take to overtake.

(*a*) 2.5 s (*b*) 7.5 s (*c*) 5 s (*d*) 1.25 s

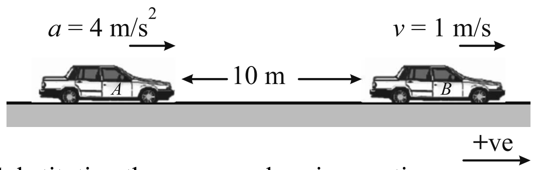
Sol. Given: $u_A = 0$, $u_B = 1$ m/s, $a_A = 4$ m/s^2 and $a_B = 0$

Assuming car B to be at rest, we have

$u_{AB} = u_A - u_B = 0 - 1 = -1$ m/s

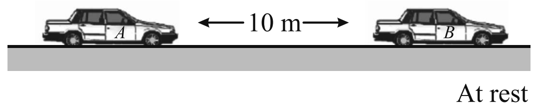
$a_{AB} = a_A - a_B = 4 - 0 = 4$ m/s^2

Now, the problem can be assumed in simplified form as follow:



Substituting the proper values in equation

$u_{AB} = -1$ m/s, $a_{AB} = 4$ m/s^2



$$S = ut + \frac{1}{2}at^2$$

we get $10 = -t + \frac{1}{2}(4)(t^2)$ or $2t^2 - t - 10 = 0$

Ignoring the negative value, the desired time is 2.5s.

Therefore, option (*a*) is the correct answer.

Note: The above problem can also be solved without using the concept of relative motion as under. At the time when A overtakes B,

$S_A = S_B + 10$

$$\therefore \frac{1}{2} \times 4 \times t^2 = 1 \times t + 10$$

or $2t^2 - t - 10 = 0$

which on solving gives $t = 2.5$ s and -2 s, the same as we found above.

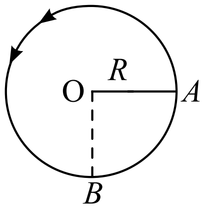
POSITION, DISTANCE AND DISPLACEMENT

1. A body moves 6 m north. 8 m east and 10 m vertically upwards, what is its resultant displacement from initial position?

(*a*) $10\sqrt{2}$ m (*b*) 10 m

(*c*) $\frac{10}{\sqrt{2}}$ m (*d*) 20 m

2. A body moves in circular path of radius *R* from *A* to *B* as shown. Its displacement and distance covered are



(*a*) $R, \frac{3\pi R}{2}$ (*b*) $\sqrt{2}R, \frac{\pi R}{2}$

(*c*) $\sqrt{2}R, \frac{3\pi R}{2}$ (*d*) None of these

3. A particle covers half of the circle of radius *r*. Then the displacement and distance of the particle are respectively

(*a*) $2\pi r$, 0 (*b*) $2r$, πr (*c*) $\frac{\pi r}{2}$, $2r$ (*d*) πr, r

SPEED AND VELOCITY

4. A person travels along a straight road for half the distance with velocity v_1 and the remaining half distance with velocity v_2. The average velocity is given by

(*a*) $v_1 v_2$ (*b*) $\frac{v_2^2}{v_1^2}$

(*c*) $\frac{v_1 + v_2}{2}$ (*d*) $\frac{2v_1v_2}{v_1 + v_2}$

5. A car travels the first half of a distance between two places at a speed of 30 km/hr and the second half of the distance at 50 km/hr. The average speed of the car for the whole journey is

(*a*) 42.5 km/hr (*b*) 40.0 km/hr

(*c*) 37.5 km/hr (*d*) 35.0 km/hr

6. A person travels along a straight road for the first half time with a velocity v_1 and the next half time with a velocity v_2. The mean velocity *V* of the man is

(*a*) $\frac{2}{V} = \frac{1}{v_1} + \frac{1}{v_2}$ (*b*) $V = \frac{v_1 + v_2}{2}$

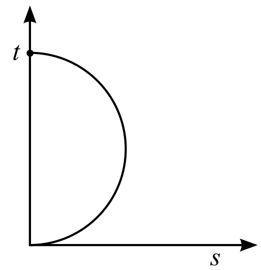
(*c*) $V = \sqrt{v_1 v_2}$ (*d*) $V = \sqrt{\frac{v_1}{v_2}}$

7. If a car covers $2/5^{th}$ of the total distance with v_1 speed and $3/5^{th}$ distance with v_2 then average speed is

(*a*) $\frac{1}{2}\sqrt{v_1 v_2}$ (*b*) $\frac{v_1 + v_2}{2}$

(*c*) $\frac{2v_1v_2}{v_1 + v_2}$ (*d*) $\frac{5v_1v_2}{3v_1 + 2v_2}$

8. Which of the following options is correct for the object having a straight line motion represented by the following graph?



(*a*) The object moves with constantly increasing velocity from *O* to *A* and then it moves with constant velocity

(*b*) Velocity of the object increases uniformly

(*c*) Average velocity is zero

(*d*) The graph shown is impossible

CONSTANT ACCELERATION

9. A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance S_1 in the first 10 sec and a distance S_2 in the next 10 sec, then

(*a*) $S_1 = S_2$ (*b*) $S_1 = S_2/3$

(*c*) $S_1 = S_2/2$ (*d*) $S_1 = S_2/4$

10. A body is moving from rest under constant acceleration and let S_1 be the displacement in the first $(p - 1)$ sec and S_2 be the displacement in the first *p* sec. The displacement in $(p^2 - p + 1)^{th}$ sec. will be

(*a*) $S_1 + S_2$ (*b*) $S_1 S_2$

(*c*) $S_1 - S_2$ (*d*) S_1 / S_2

11. The displacement of body moving with constant acceleration, in 3rd seconds is 2m and in 5th second is 9m. Find the acceleration of body.

(*a*) $\frac{5}{2}$ ms^{-2} (*b*) $\frac{7}{2}$ ms^{-2}

(*c*) $\frac{9}{2}$ ms^{-2} (*d*) $\frac{11}{2}$ ms^{-2}

12. A point moves with uniform acceleration and v_1, v_2 and v_3 denote the average velocities in the three successive intervals of time t_1, t_2 and t_3. Which of the following relations is correct?

(*a*) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$
(*b*) $(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$
(*c*) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 - t_3)$
(*d*) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$

13. Which of the following four statements is false?
(*a*) A body can have zero velocity and still be accelerated.
(*b*) A body can have a constant velocity and still have a varying speed.
(*c*) A body can have a constant speed and still have a varying velocity.
(*d*) The direction of the velocity of a body can change when its acceleration is constant.

14. A car starting from rest and moving with acceleration of $4ms^{-2}$, covers half the distance during last second of motion before it strikes a vertical wall. Find the distance of wall from starting point.
(*a*) 23.3 m (*b*) 24 m
(*c*) 24.3 m (*d*) 26.3 m

15. A car starts from rest and moves with uniform acceleration a on a straight road from time $t = 0$ to $t = T$. After that, constant deceleration brings it to rest. In this process the average speed of the car is
(*a*) $\frac{aT}{4}$ (*b*) $\frac{3aT}{2}$ (*c*) $\frac{aT}{2}$ (*d*) aT

16. A car, starting from rest, accelerates at constant rate f through a distance S, then continues at constant speed for time t and thaen decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then
(*a*) $S = \frac{1}{2}ft^2$ (*b*) $S = \frac{1}{4}ft^2$
(*c*) $S = \frac{1}{72}ft^2$ (*d*) $S = \frac{1}{6}ft^2$

MOTION UNDER GRAVITY

17. A stone falls from a balloon that is descending at a uniform rate of 12 m/s. The displacement of the stone from the point of release after 10 sec is
(*a*) 490 m (*b*) 510 m (*c*) 610 m (*d*) 725 m

18. Two bodies of different masses m_a and m_b are dropped from two different heights a and b. The ratio of the time taken by the two to cover these distances are
(*a*) $a : b$ (*b*) $b : a$ (*c*) $\sqrt{a} : \sqrt{b}$ (*d*) $a^2 : b^2$

19. Two balls are projected simultaneously with the same speed from the top of a tower, one vertically upwards and the other vertically downwards. If the first ball strikes the ground with speed 20 m/s then speed of second ball when it strikes the ground is
(*a*) 10 m/s (*b*) 20 m/s
(*c*) 40 m/s (*d*) Data insufficient

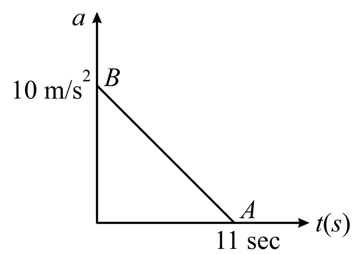
20. A ball is released from the top of a tower of height h. It takes t sec to reach the ground. Where will be the ball after time $t/2$ sec?
(*a*) At $h/2$from the ground
(*b*) At $h/4$ from the ground
(*c*) Depends upon mass and volume of the ball
(*d*) At $3h/4$ from the ground

21. A body is slipping from an inclined plane of height h and length l. If the angle of inclination is θ, the time taken by the body to come from the top to the bottom of this inclined plane is
(*a*) $\sqrt{\frac{2h}{g}}$ (*b*) $\sqrt{\frac{2h}{g}}$
(*c*) $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$ (*d*) $\sin\theta\sqrt{\frac{2h}{g}}$

22. A man in a balloon rising vertically with an acceleration of 4.9 m/sec^2 releases a ball 2 sec after the balloon is let go from the ground. The greatest height above the ground reached by the ball is ($g = 9.8 \text{ m/sec}^2$)
(*a*) 14.7 m (*b*) 19.6 m (*c*) 9.8 m (*d*) 24.5 m

VARIABLE ACCELERATION

23. A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be



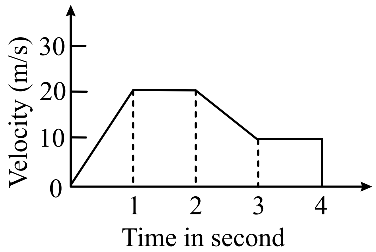
(*a*) 110 m/s (*b*) 55 m/s
(*c*) 550 m/s (*d*) 660 m/s

24. If the velocity of a particle is given by $v = (180 - 16x)^{1/2}$ m/s, then its acceleration will be
(*a*) Zero (*b*) 8 m/s^2 (*c*) -8 m/s^2 (*d*) 4 m/s^2

25. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will
(*a*) Go on decreasing with time
(*b*) Will be independent of α and β
(*c*) Drop to zero when α = β
(*d*) Go on increasing with time

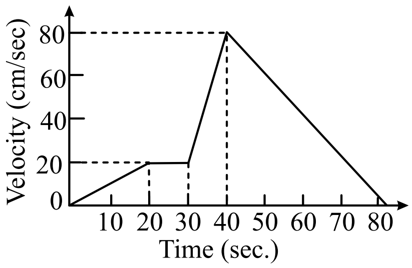
GRAPHS

26. The variation of velocity of a particle with time moving along a straight line is illustrated in the following figure. The distance travelled by the particle in four seconds is



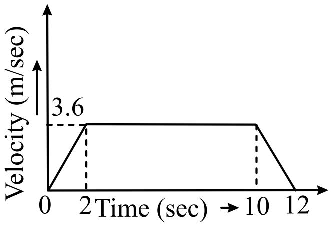
(a) 60 m (b) 55 m (c) 25 m (d) 30 m

27. The $v - t$ graph of a moving object is given in figure. The maximum acceleration is



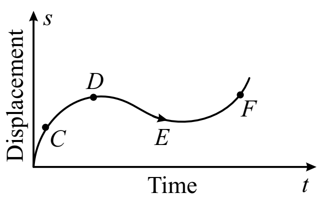
(a) 1 cm/sec^2 (b) 2 cm/sec^2
(c) 3 cm/sec^2 (d) 6 cm/sec^2

28. A lift is going up. The variation in the speed of the lift is as given in the graph. What is the height to which the lift takes the passengers?



(a) 3.6 m
(b) 28.8 m
(c) 36.0 m
(d) Cannot be calculated from the above graph

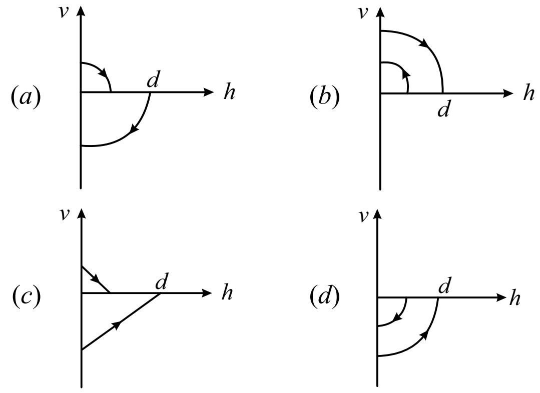
29. The displacement-time graph of moving particle is shown below



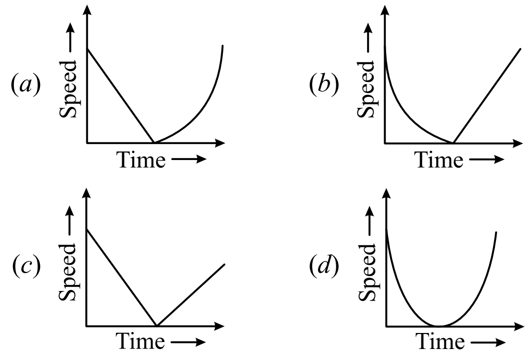
The instantaneous velocity of the particle is negative at the point.

(a) D (b) F (c) C (d) E

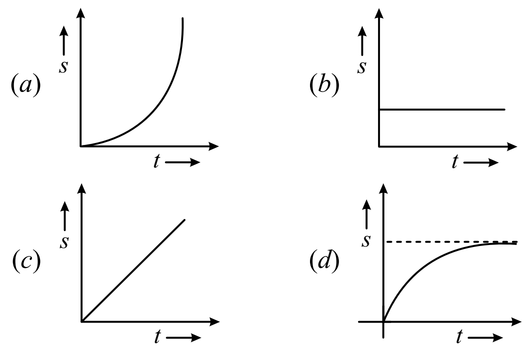
30. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as



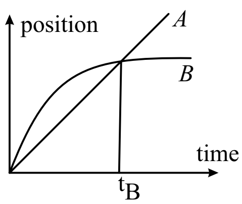
31. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its motion if the air resistance (constant) is not ignored?



32. Which graph must represent non-uniform acceleration (s is displacement)?



33. The graph shows position as a function of time for two trains running on parallel tracks. Which one of the following statements is true?



(a) At time t_B, both trains have the same velocity
(b) Both trains have the same velocity at some time after t_B
(c) Both trains have the same velocity at some time before t_B
(d) Somewhere on the graph, both trains have the same acceleration

ONE DIMENSIONAL RELATIVE MOTION

34. Two trains, each 50 m long are travelling in opposite direction with velocity 10 m/s and 15 m/s. The time of crossing is

(a) 2 s (b) 4 s (c) $2\sqrt{3}$ s (d) $4\sqrt{3}$ s

35. A 210 meter long train is moving due North at a of 25m/s. A small bird is flying due South a little above the train with speed 5m/s. The time taken by the bird to cross the train is

(*a*) 6 *s* (*b*) 7 *s* (*c*) 9 *s* (*d*) 10 *s*

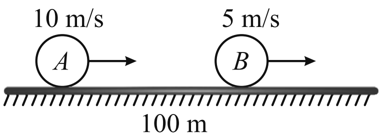
36. A stone is dropped from a building, and 2 seconds later another stone is dropped. (Both a are dropped from rest.) How far apart are the two stones by the time the first one has reached a speed of 30 m/s ?

(*a*) 80 m (*b*) 100 m (*c*) 60 m (*d*) 40 m

37. Two trains each of length 50 m are approaching each other on parallel rails. Their velocities are 10 m/sec and 15 m/sec. They will cross each other in

(*a*) 2 sec (*b*) 4 sec (*c*) 10 sec (*d*) 6 sec

38. An object A is moving with 10 m/s and B is moving with 5 m/s in the same direction of positive x-axis. A is 100 m behind B as shown. Find time taken by A to Meet B



(*a*) 18 sec (*b*) 16 sec

(*c*) 20 sec (*d*) 17 sec

39. A thief is running away on a straight road with a speed of 9 ms^{-1}. A police man chases him on a jeep moving at a speed of 10 ms^{-1}. If the instantaneous separation of the jeep from the motorcycle is 100m, how long will it take for the police man to catch the thief?

(*a*) 1s (*b*) 19s

(*c*) 90s (*d*) 100s

DISTANCE OF NEAREST APPROACH

40. A body is projected vertically up at $t = 0$ with a velocity of 98 m/s. Another body is projected from the same point with same velocity after 4 seconds. Both bodies will meet after:

(*a*) 6 *s* (*b*) 8 *s*

(*c*) 10 *s* (*d*) 12 *s*

PRABAL (JEE MAIN LEVEL)

1. A car runs at constant speed on a circular track of radius 100 m taking 62.8 s on each lap. What is the average speed and average velocity on each complete lap?

(*a*) Average velocity 10 m/s, average speed 10 m/s

(*b*) Average velocity zero, average speed 10 m/s

(*c*) Average velocity zero, average speed zero

(*d*) Average velocity 10 m/s, average speed zero

2. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10s is x_1, in next 10 s is x_2 and in last 10 s is x_3. Then $x_1 : x_2 : x_3$ is

(*a*) 1 : 2 : 4 (*b*) 1 : 2 : 5

(*c*) 1 : 3 : 5 (*d*) 1 : 3 : 9

3. A body is thrown upward and reaches its maximum height. At that position

(*a*) Its velocity is zero and its acceleration is also zero

(*b*) Its velocity is zero but its acceleration is maximum

(*c*) Its acceleration is minimum

(*d*) Its velocity is zero and its acceleration is the acceleration due to gravity

4. The motion of a body is given by the equation $\frac{dv(t)}{dt} = 6.0 - 3v(t)$, where $v(t)$ is speed in m/s and t in sec.

If body was at rest at $t = 0$ choose the wrong option.

(*a*) The terminal speed is 2.0 m/s

(*b*) The speed varies with the time as $v(t) = 2(1 - e^{-3t})$ m/s

(*c*) The speed is 0.1 m/s when the acceleration is half the initial value

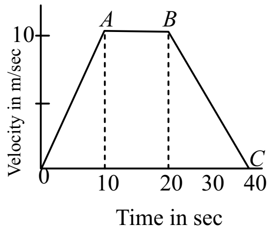
(*d*) The magnitude of the initial acceleration is 6.0 m/s^2

5. The displacement time graphs of motion of two particles A and B are straight lines making angles of 30º and 60º respectively with the time axis. If the velocity of A is v_A and that of B is v_B then the value of $\frac{v_A}{v_B}$ is

(*a*) 1/2 (*b*) $1/\sqrt{3}$

(*c*) $\sqrt{3}$ (*d*) 1/3

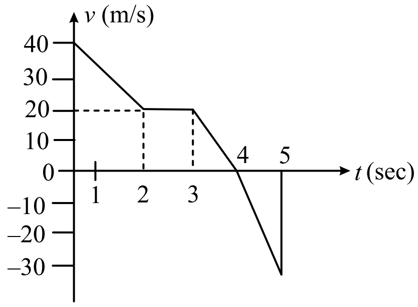
6. The curve shown represents the velocity-time graph of a particle, its acceleration values along OA, AB and BC in metre/sec^2 are respectively



(*a*) 1, 0, –0.5 (*b*) 1, 0, 0.5

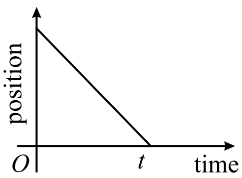
(*c*) 1, 1, 0.5 (*d*) 1, 0.5, 0

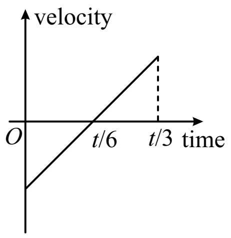
7. In the following velocity-time graph of a body, the distance and displacement travelled by the body in 5 second in meters will be

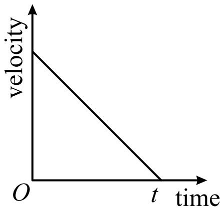


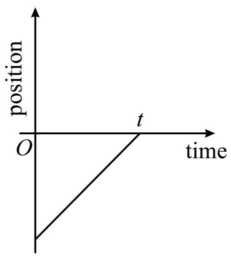
(a) 75, 115 (b) 105, 75
(c) 45, 75 (d) 95, 55

8. For which of the following graphs the average velocity of a particle moving along a straight line for time interval $(0, t)$ must be negative?

(a)


(b)


(c)


(d)


9. Four particles move along x-axis. Their coordinates (in meters) as functions of time (in seconds) are given by

Particle 1 : $x(t) = 3.5 - 2.7t^3$

Particle 2 : $x(t) = 3.5 + 2.7t^3$

Particle 3 : $x(t) = 3.5 + 2.7t^2$

Particle 4 : $x(t) = 3.5 - 3.4t - 2.7t^2$

Which of these particles have constant acceleration?

(a) All four (b) Only 1 and 2
(c) Only 2 and 3 (d) Only 3 and 4

10. A particle is projected up from ground with initial speed v_0. Starting from time $t = 0$ to $t = t_1$,

(a) Distance travelled and magnitude of displacement are not equal if $t_1 < \frac{v_0}{g}$

(b) Distance travelled and magnitude of displacement are equal if $\frac{v_0}{g} < t_1 < \frac{2v_0}{g}$

(c) Distance travelled and magnitude of displacement may not be equal if $0 < t_1 < \frac{2v_0}{g}$

(d) The magnitude of displacement is greater than the distance travelled if $\frac{v_0}{g} < t_1 < \frac{2v_0}{g}$

11. Two bodies P and Q have to move equal distances starting from rest. P is accelerated with $2a$ for first half distance, then its acceleration becomes a for last half, whereas Q has acceleration a for first half and acceleration $2a$ for last half, then for whole journey.

(a) Average speed of P is more than that of Q

(b) Average speed of both will be same

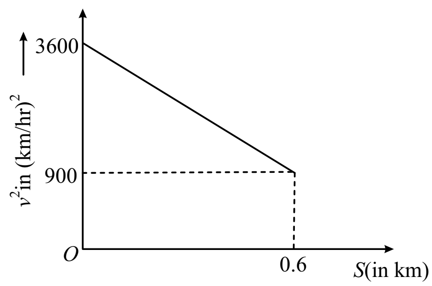
(c) Maximum speed during the journey is more for P

(d) Maximum speed during the journey is more for Q

12. A car is moving along a straight line. It is taken from rest to a velocity of 20 ms^{-1} by a constant acceleration of 5ms^{-2}. It maintains a constant velocity of 20 ms^{-1} for 5 seconds and then is brought to rest again by a constant acceleration of –2 ms^{-2}. Find the distance covered by the car.

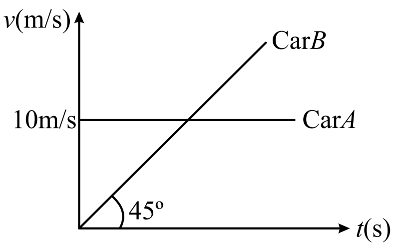
(a) 120 m (b) 200 m
(c) 240 m (d) 400 m

13. A graph between the square of the velocity of a particle and the distance 'S' moved by the particle is shown in the figure. The acceleration of the particle in kilometer per hour square is



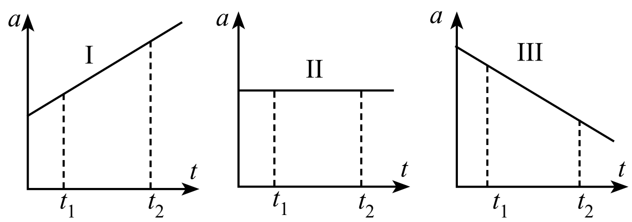
(a) 2250 (b) 225
(c) –2250 (d) –225

14. Initially car A is 10.5 m ahead of car B. Both start moving at time $t = 0$ in the same direction along a straight line. The velocity time graph of two cars is shown in figure. The time when the car B will catch the car A, will be



(a) $t = 21$ s (b) $t = 2\sqrt{5}$ s
(c) $t = 20$ s (d) None of these

15. Each of the three graphs represents acceleration versus time for an object that already has a positive velocity at time t_1. Which graphs show an object whose speed is increasing for the entire time interval between t_1 and t_2?



(a) graph I, only (b) graphs I and II, only

(c) graphs I and III, only (d) graphs I, II, and III

16. From the top of a tower, a stone is thrown up. It reaches the ground in time t_1. *A* second stone thrown down with the same speed reaches the ground in time t_2. A third stone released from rest reaches the ground in time t_3. Then:

(a) $t_3 = \dfrac{t_1 + t_2}{2}$ (b) $t_3 = \sqrt{t_1 t_2}$

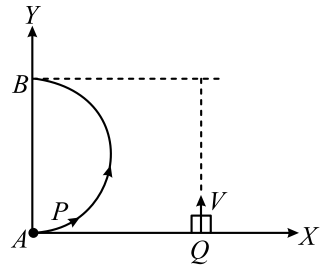
(c) $\dfrac{1}{t_3} = \dfrac{1}{t_1} - \dfrac{1}{t_2}$ (d) $t_3^2 = t_1^2 - t_2^2$

17. A bus is moving with a velocity 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100s. If the bus is at a distance of 1 km from the scooterist, with what velocity should the scooterist chase the bus?

(a) 50 ms^{-1} (b) 40 ms^{-1}

(c) 30 ms^{-1} (d) 20 ms^{-1}

18. A particle P starts from origin as shown and moves along a circular path. Another particle Q crosses x-axis at the instant particle P leaves origin. Q moves with constant speed V parallel to y-axis and is all the time having y-coordinate same as that of P. When P reaches diametrically opposite at point B, its average speed is



(a) πV (b) $\dfrac{\pi V}{2}$

(c) $\dfrac{V}{2}$ (d) None of these

INTEGER TYPE QUESTIONS

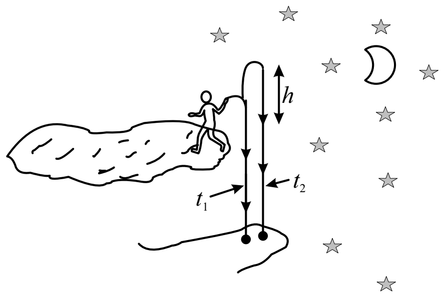
19. A thief in a stolen car passes through a police check post at his top speed of 90 kmh^{-1}. A motorcycle cop, reacting after 2 s, accelerates from rest at 5 ms^{-2}. His top speed being 108 kmh^{-1}. Find the maximum separation between policemen and thief is $K \times 10^{-1}$m. Find K.

20. Two trains are moving with velocities v_1 = 10 ms^{-1} and v_2 = 20 ms^{-1} on the same track in opposite directions. After the application of brakes if their retarding rates are a_1 = 2 ms^{-2} and a_2 = 1 ms^{-2} respectively, then find the minimum distance of separation (in m) between the trains to avoid collision.

21. Two trains, each travelling with a speed of 37.5 kmh^{-1}, are approaching each other on the same straight track. A bird that can fly at 60 kmh^{-1} flies off from one train when they are 90 km apart and heads directly for the other train. On reaching the other train, it flies back to the first and so on. Total distance covered by the bird is _______ km.

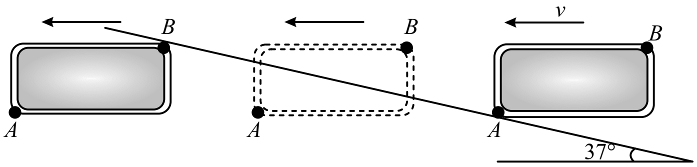
22. A large procession of people is moving along a road of width 10 m. There is a railway track across the road. The number of people present per square meter of the road is 6 (on an average). The average speed at which the procession is moving is 0.4 m/s. Find the number of people crossing the railway track per second.

23. An astronaut on the starship Enterprise is roaming around on a distant planet. He drops a rock from the top of a cliff and observes that it takes time $t_1 = \sqrt{2}$ sec to reach the bottom. He now throws another rock vertically upwards so that it reaches a height h = 10 m above the cliff before dropping down the cliff. The second rock takes a total time t_2 = 2 sec to reach the bottom of the cliff, starting from the time it leaves the astronaut's hand. The planet has a very thin atmosphere which offers negligible air resistance. What is the value of acceleration (in m/s^2) due to gravity on this planet?



24. A stone is dropped from a certain height and can reach the ground in 5s. But in its fall, the stone is stopped after 3s of the fall for a moment and is dropped again at once. Now the stone reaches the ground in total time of 't' seconds. Find the value of t.

25. A train is travelling at v m/s along a level straight track. Very near and parallel to the track is a wall. On the wall a naughty boy has drawn a straight line that slopes upward at 37° angle with the horizontal. A passenger in the train is observing the line out of window (0.90 m high, 1.8 m wide as shown in figure). The line first appears at window corner A and finally disappears at window corner B. If it takes 0.4 sec between appearance at A and disappearance of the line at B, what is the value of v (in cm/s)?



PARIKSHIT (JEE ADVANCED LEVEL)

MULTIPLE CORRECT TYPE QUESTIONS

1. Mark the correct statements for a particle going on a straight line

(*a*) If the velocity is zero at any instant, the acceleration should also be zero at that instant.

(*b*) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.

(*c*) If the velocity and acceleration have opposite sign, the object is slowing down.

(*d*) If the position and velocity have opposite sign, the particle is moving towards the origin.

2. Let $\vec{v}$ and $\vec{a}$ denote the velocity and acceleration respectively of a body in one-dimensional motion then which among the following is/are true?

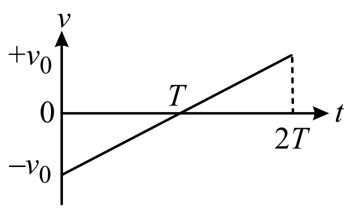
(*a*) $|\vec{v}|$ must decrease when $\vec{a} < 0$.

(*b*) Speed must increase when $\vec{a} > 0$.

(*c*) Speed will increase when both $\vec{v}$ and $\vec{a}$ are < 0.

(*d*) Speed will decrease when $\vec{v} < 0$ and $\vec{a} > 0$.

3. The figure shows the velocity (v) of a particle plotted against time (t)



(*a*) The particle changes its direction of motion at some point.

(*b*) The acceleration of the particle remains constant.

(*c*) The displacement of the particle is zero.

(*d*) The initial and final speeds of the particle are the same.

4. A particle moves with constant speed v along a regular hexagon *ABCDEF* in the same order. Then the magnitude of the average velocity for its motion from A to

(*a*) F is $v/5$ (*b*) D is $v/3$

(*c*) C is $\frac{v\sqrt{3}}{2}$ (*d*) B is v

5. Path of a particle moving in x-y plane is $y = 3x + 4$. At some instant suppose x-component of velocity is 1 m/s and it is increasing at a rate of 1 m/s^2. Then

(*a*) At this instant the speed of particle is $\sqrt{10}$ m/s.

(*b*) At this instant the acceleration of particle is $\sqrt{10}$ m/s^2.

(*c*) Velocity time graph is a straight line.

(*d*) Acceleration-time graph is a straight line.

6. A particle having a velocity $v = v_0$ at $t = 0$ is brought to rest by decelerating at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant.

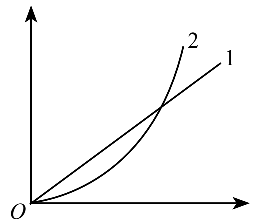
(*a*) The particle comes to rest at $t = \frac{2\sqrt{v_0}}{\alpha}$

(*b*) The particle will come to rest at infinity.

(*c*) The distance travelled by the particle is $\frac{2v_0^{3/2}}{\alpha}$.

(*d*) The distance travelled by the particle is $\frac{2}{3}\frac{v_0^{3/2}}{\alpha}$.

7. A particle is resting over a smooth horizontal floor. At $t = 0$, a horizontal force starts acting on it. Magnitude of the force increases with time as $F = kt$, where k is a constant. Two curves are drawn for this particle as shown.



(*a*) Curve-1 shows acceleration versus time.

(*b*) Curve-2 shows velocity versus time.

(*c*) Curve-2 shows velocity versus acceleration.

(*d*) Curve-1 shows velocity versus acceleration.

8. The minimum speed with respect to air that a particular jet aircraft must have in order to keep aloft is 300 km/hr. Suppose that as its pilot prepares to take off, the wind blows eastward at a ground speed that can vary between 0 and 30 km/hr. Ignoring any other fact, a safe procedure to follow, consistent with using up as little fuel as possible, is to:

(*a*) Take off eastward at a ground speed of 320 km/hr

(*b*) Take off westward at a ground speed of 320 km/hr

(*c*) Take off westward at a ground speed of 300 km/hr

(*d*) Take off westward at a ground speed of 280 km/hr

9. A block is thrown with a velocity of 2 ms^{-1} (relative to ground) on a belt, which is moving with velocity 4 ms^{-1} in opposite direction of the initial velocity of block. If the block stops slipping on the belt after 4 sec of the throwing then choose the correct statements

(*a*) Displacement with respect to ground is zero after 2.66s and magnitude of displacement with respect to ground is 12 m after 4 sec.

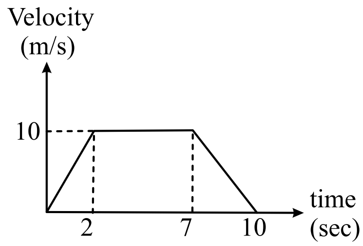
(*b*) Magnitude of displacement with respect to ground in 4 sec is 4 m.

(*c*) Magnitude of displacement with respect to belt in 4 sec is 12 m.

(*d*) Displacement with respect to ground is zero in 8/3 sec

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 10 to 12): The velocity-time graph of a car moving on a straight track is given below. The car weighs 1000 kg. (Use $F = ma$)



10. The distance travelled by the car during the whole motion is

(*a*) 50 m (*b*) 75 m (*c*) 100 m (*d*) 150 m

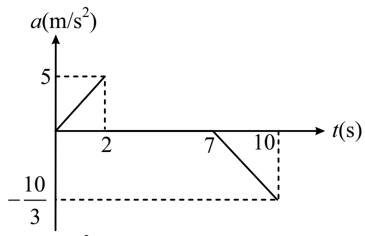
11. The braking force required to bring the car to a stop within one second from the maximum speed is

(*a*) $\frac{10000}{3}$ N (*b*) 5000 N

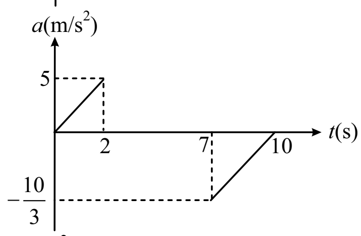
(*c*) 10000 N (*d*) $\frac{5000}{3}$ N

12. Correct acceleration-time graph representing the motion of car is

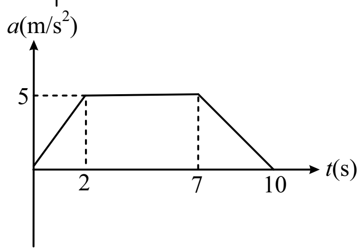
(*a*)



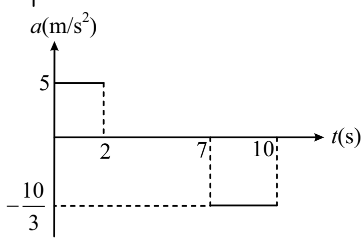
(*b*)



(*c*)

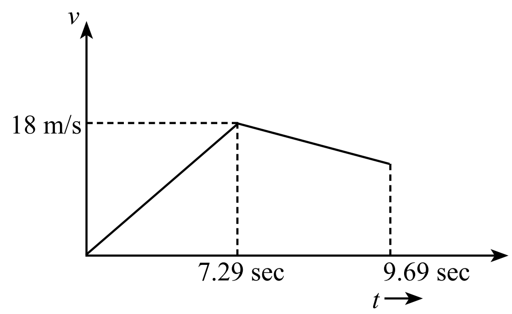


(*d*)



Comprehension (Q. 13 to 14): In the 2008 Olympic 100 m final, Usain Bolt broke new ground, winning in 9.69 s (unofficially 9.683 s). This was an improvement upon his own world record, and he was well ahead of second-place finisher Richard Thompson, who finished in 9.89 s. Not only was the record set without a favourable wind (+0.0 m/s), but he also visibly slowed down to celebrate before he finished and his shoelace was untied. Bolt's coach reported that, based upon the speed of Bolt's opening, he could have finished with a time of 9.52 s. After scientific analysis of Bolt's run by the Institute of Theoretical Astrophysics at the University of Oslo, Hans Eriksen and his colleagues also predicted a 9.60 s time. Considering factors such as Bolt's position, acceleration and velocity in comparison with second-place-finisher Thompson, the team estimated that Bolt could have finished in 9.55 ± 0.04 s had he not slowed to celebrate before the finishing line.

Let us also analyse the motion of Bolt. Assume that the velocity time graph of Usain Bolt is as shown below.



13. What was the initial acceleration of Bolt?

(*a*) 4.5 m/s^2 (*b*) 3.1 m/s^2

(*c*) 2.5 m/s^2 (*d*) 1.2 m/s^2

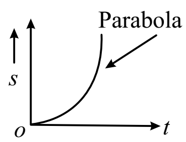
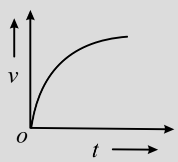
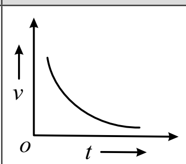
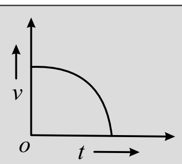
14. What was the final velocity of Bolt?

(*a*) 10.1 m/s (*b*) 10.6 m/s

(*c*) 13.4 m/s (*d*) 14.6 m/s

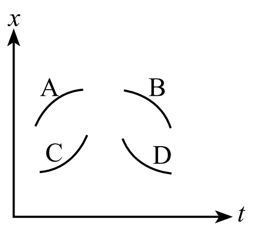
MATCH THE COLUMN TYPE QUESTIONS

15. Match Column-I with Column-II and select the correct answer using the codes given below the lists.

	Column-I		Column-II
A.	Acceleration decreasing with time	p.	
B.	Velocity increasing with time	q.	
C.	Magnitude of acceleration increasing with time	r.	
D.	Body going farther away from the starting point with time	s.	

(*a*) A-(q,s); B-(p,q); C-(s); D-(p,q,r,s)
(*b*) A-(r,s); B-(r,q); C-(p,q,r,s); D-(s)
(*c*) A-(r,q); B-(q); C-(s,r); D-(p,q,r,s)
(*d*) A-(r,s); B-(p,r); C-(p,q,r,s); D-(p,q)

16. Column-I shows the position-time graph of particles moving along a straight line and column-II lists the conclusion that follow from graphs. Match column-I with column-II and choose the correct option given below the columns.

Column-I	Column-II	
	p.	Acceleration $a > 0$
	q.	Acceleration $a < 0$
	r.	Speeding up
	s.	Slowing down

(*a*) A-(q,p); B-(r,p); C-(s,q); D-(r,s)
(*b*) A-(q,s); B-(q,r); C-(p,r); D-(p,s)
(*c*) A-(r,q); B-(s,r); C-(q,r); D-(p,r)
(*d*) A-(p,s); B-(r,s); C-(p,r); D-(p,q)

17. The position of a particle along x-axis is given by $x = (2t^3 - 21t^2 + 60t)m$. Then match the Column-I with Column-II.

Column-I		Column-II	
A.	Velocity of particle is zero	p.	2 sec
B.	Acceleration of particle is zero	q.	3 sec
C.	Acceleration of particle is negative	r.	3.5 sec
D.	Velocity of particle is towards the origin	s.	4 sec
		t.	5 sec

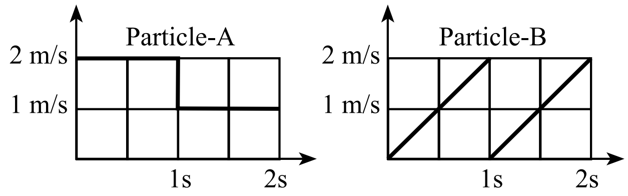
(*a*) A-(p,r,t); B-(r); C-(p,q); D-(q,r,s)
(*b*) A-(p,t,r); B-(r,t); C-(q); D-(s)
(*c*) A-(p,r); B-(s); C-(p,r,q); D-(r,s)
(*d*) A-(p,t); B-(r); C-(p,q); D-(q,r,s)

NUMERICAL TYPE QUESTIONS

Answer should be rounded off upto two decimal places

18. A particle is thrown upwards from ground. It experiences a constant resistance force which can produce retardation 2 m/s^2. The ratio of time of ascent to the time of descent is____. [g = 10 m/s^2]

19. Two particles A and B start from the same point and move in the positive x-direction. Their velocity-time relationships are shown in the following figures. What is the maximum separation (in m) between them during the time interval shown?

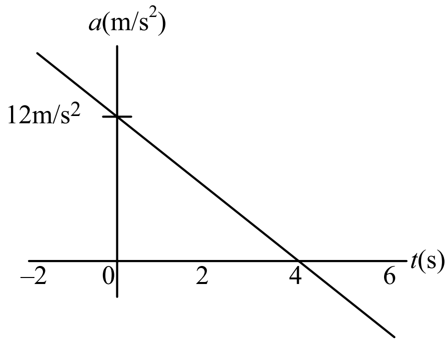


20. A train takes 2 minutes to acquire its full speed 60kmph from rest and 1 minute to come to rest from the full speed. If somewhere in between two stations 1 km of the track be under repair and the limited speed on this part be fixed to 20kmph, find the time (in s) of late running of the train on account of this repair work, assuming otherwise normal running of the train between the stations.

21. A man walking from town A to another town B at the rate of 4 km/hour starts one hour before a coach (also travelling from A to B). The coach is travelling at the rate of 12 km/hr and on the way he is picked up by the coach. On arriving at B, he finds that his coach journey lasted 2 hours. Find the distance (in km) between A and B.

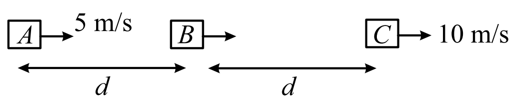
INTEGER TYPE QUESTIONS

22. Figure gives the acceleration a versus time t for a particle moving along an x-axis. At t = –2.0 s, the particle's velocity is 7.0 m/s. What is its velocity (in m /s) at t = 6.0 s?



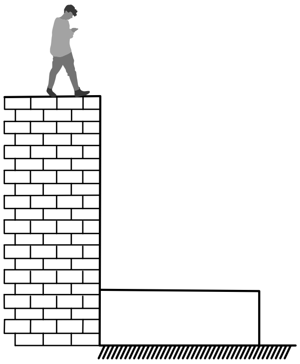
23. A particle moves in xy-plane according to the equation $x = 3t$, $y = 25 - 4t$. What is the minimum distance of the particle (in m) from the origin? Both x and y are in m.

24. Three persons A, B, C are moving along a straight line, as shown, with constant but different speeds. When B catches C, the separation between A and C becomes $4d$, then the speed of B is u. Find $2u$.



SINGLE CORRECT TYPE QUESTIONS

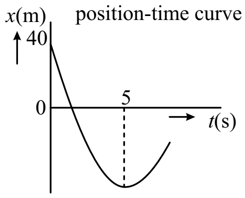
25. Suppose that a man jumps off a building 202 m high onto cushions having a total thickness of 2 m. If the cushions are crushed to a thickness of 0.5 m, what is the man's acceleration (assumed constant) as he slows down?

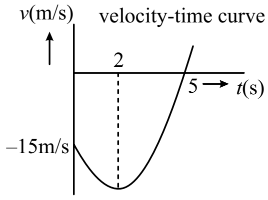


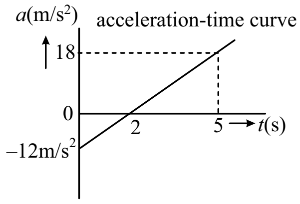
(a) 10 m/s^2 (b) $\frac{4000}{3}$ m/s^2

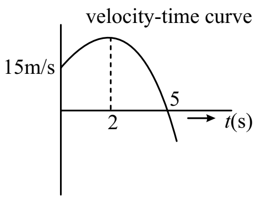
(c) 50 m/s^2 (d) 20 m/s^2

26. The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in meters and t in seconds. Which of the graph does not represent the motion of the particle?

(a) 

(b) 

(c) 

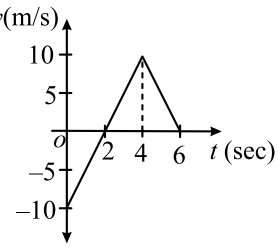
(d) 

27. A trolley is moving away from a stop with an acceleration a = 0.2 m/s^2. After reaching the velocity u = 36 km/h, it moves with a constant velocity for the time of 2 min. Then, it uniformly slows down, and stops after further travelling a distance of 100 m. Find the average speed all the way between stops.

(a) $\frac{76}{17}$ m/s (b) $\frac{208}{21}$ m/s

(c) $\frac{85}{12}$ m/s (d) $\frac{155}{19}$ m/s

28. The figure shows the graph of velocity-time for a particle moving in a straight line. If the average speed for 6 sec is 'b' and the average acceleration from 0 sec to 4 sec is 'c' find magnitude of bc (in m^2/s^3).



(a) 5 (b) 20

(c) 25 (d) 40

29. James bond is standing on a bridge above the road below and his pursuers are getting too close for comfort. He spots a flat bed truck loaded with mattresses approaching at 30 m/s which he measures by knowing that the telephones poles the truck is passing are 20m apart in this country. The bed of truck is 20m below the bridge and bond quickly calculates how many poles away the truck should be when he jumps down the bridge onto the truck making his get away. How many poles is it ?

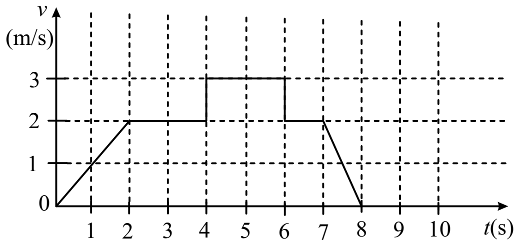
(a) 3 (b) 4

(c) 5 (d) 6

PYQ's (PAST YEAR QUESTIONS)

POSITION, DISTANCE AND DISPLACEMENT

1. A particle starts from the origin at time $t = 0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5s$? **[10 Jan, 2019 (Shift-II)]**



(*a*) 10 m (*b*) 6 m (*c*) 3 m (*d*) 9 m

SPEED AND VELOCITY

2. A horse rider covers half the distance with 5 m/s speed. The remaining part of the distance was travelled with speed 10 m/s for half the time and with speed 15 m/s for other half of the time. The mean speed of the rider averaged over the whole time of motion is $x/7$ m/s. The value of x is ________ .

[30 Jan, 2023 (Shift-I)]

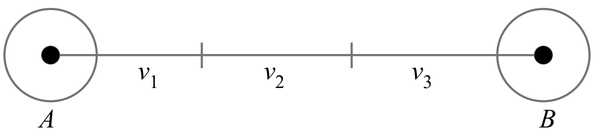
3. A car travels a distance of 'x' with speed v_1 and then same distance 'x' with speed v_2 in the same direction. The average speed of the car is: **[25 Jan, 2023 (Shift-I)]**

(*a*) $\dfrac{v_1 v_2}{2(v_1+v_2)}$ (*b*) $\dfrac{v_1+v_2}{2}$

(*c*) $\dfrac{2x}{v_1+v_2}$ (*d*) $\dfrac{2v_1v_2}{v_1+v_2}$

4. A car covers AB distance with first one-third at velocity v_1 ms^{-1}, second one-third at v_2 ms^{-1} and last one-third at v_3 ms^{-1}. If $v_3 = 3v_1$, $v_2 = 2v_1$ and $v_1 = 11$ ms^{-1} then the average velocity of the car is ________ms^{-1}.

[28 June, 2022 (Shift-II)]



5. A car is moving with speed of 150 km/h and after applying the break it will move 27m before it stops. If the same car is moving with a speed of one third the reported speed then it will stop after travelling __________ m distance.

[25 July, 2022 (Shift-I)]

6. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$; then its displacement after time ($t = 1$) is: **[17 March, 2021 (Shift-II)]**

(*a*) $v_0 + g + f$ (*b*) $v_0 + \dfrac{g}{2} + \dfrac{F}{3}$

(*c*) $v_0 + 2g + 3F$ (*d*) $v_0 + \dfrac{g}{2} + F$

7. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be: **[9 April, 2019 (Shift-II)]**

(*a*) $a + \dfrac{b^2}{4c}$ (*b*) $a + \dfrac{b^2}{c}$

(*c*) $a + \dfrac{b^2}{2c}$ (*d*) $a + \dfrac{b^2}{3c}$

CONSTANT ACCELERATION

8. For a train engine moving with speed of 20 ms^{-1}. the driver must apply brakes at a distance of 500m before the station for the train to come to rest at the station. If the brakes were applied at half of this distance, the train engine would cross the station with speed $\sqrt{x}ms^{-1}$. The value of x is ________.

(Assuming same retardation is produced by brakes)

[1 Feb, 2023 (Shift-II)]

9. A particle starts with an initial velocity of 10.0ms^{-1} along x-direction and accelerates uniformly at the rate of 2.0 ms^{-2}. The time taken by the particle to reach the velocity of 60.0 ms^{-1} is **[6 April, 2023 (Shift-II)]**

(*a*) 6s (*b*) 3s

(*c*) 30s (*d*) 25s

10. A particle is moving in a straight line such that its velocity is increasing at 5 ms^{-1} per meter. The acceleration of the particle is ________ms^{-2} at a point where its velocity is 20 ms^{-1}. **[25 July, 2022 (Shift-II)]**

11. An engine of a train, moving with uniform acceleration, passes the signal - post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is :

[25 Feb, 2021 (Shift-I)]

(*a*) $\dfrac{u+v}{2}$ (*b*) $\sqrt{\dfrac{v^2+u^2}{2}}$

(*c*) $\dfrac{v-u}{2}$ (*d*) $\sqrt{\dfrac{v^2-u^2}{2}}$

12. In a car race on straight road, car A takes a time 't' less than car B at the finish and passes finishing point with a speed 'V' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to **[9 Jan, 2019 (Shift-II)]**

(*a*) $\frac{2a_1a_1}{a_1+a_2}t$ (*b*) $\sqrt{2a_1a_2}t$

(*c*) $\sqrt{a_1a_2}\,t$ (*d*) $\frac{a_1+a_2}{2}t$

MOTION UNDER GRAVITY

13. A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3s and 5s is $\frac{x+1}{x}$. The value of x is ________ . **[12 April, 2023 (Shift-1)]**
Take (g = 10 m/s^2).

(*a*) 6 (*b*) 5 (*c*) –5 (*d*) 10

14. A tennis ball is dropped on to the floor from a height of 9.8*m*. It rebounds to a height 5.0 *m*. Ball comes in contact with the floor for 0.2 *s*. The average acceleration during contact is _________ ms^{-2} [Given g = 10 ms^{-2}]
[29 Jan, 2023 (Shift-I)]

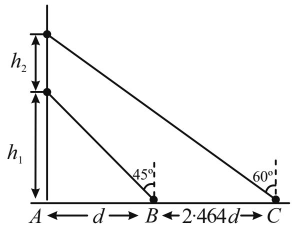
15. Two balls *A* and *B* placed at the top of 180m tall tower. Ball *A* is dropped from the top at t = 0s. Ball *B* is thrown vertically down with an initial velocity 'u' at t = 2s. After a certain time, both balls meet 100*m* above the ground. Find the value of 'u' in ms^{-1}. **[29 June, 2022 (Shift-I)]**
[use g = 10 ms^{-2}]:

(*a*) 10 (*b*) 15 (*c*) 20 (*d*) 30

16. A ball is thrown vertically upwards with a velocity of 19.6 ms^{-1} from the top of a tower. The ball strikes the ground after 6 s. The height from the ground up to which the ball can rise will be $\left(\frac{k}{5}\right)m$. The value of k is _______
(use g = 9.8 m/s^2) **[28 July, 2022 (Shift-II)]**

17. Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of _________ m. (Take g = 10 m/s^2) **[26 Aug, 2021 (Shift-I)]**

18. A balloon is moving up in air vertically above a point *A* on the ground. When it is at a height h_1, a girl standing at a distance *d* (point *B*) from *A* (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 *d* (point *C*). Then the height h_2 is (given tan 30° = 0.5774) **[5 Sep, 2020 (Shift-I)]**



(*a*) *d* (*b*) 0.732 *d*

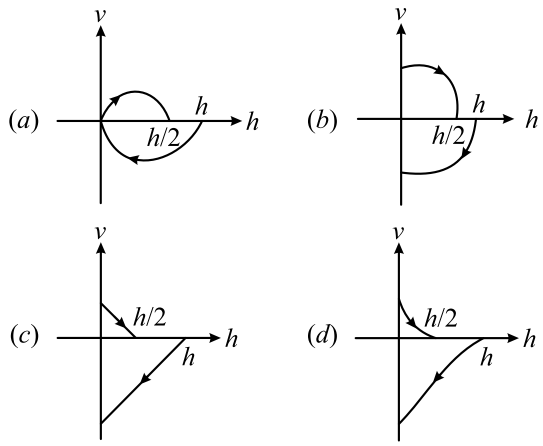
(*c*) 1.464 *d* (*d*) 0.464 *d*

19. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height *h*. The time taken by the packet to reach the ground is close to [*g* is the acceleration due to gravity] **[5 Sep, 2020 (Shift-I)]**

(*a*) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$ (*b*) $t = 1.8\sqrt{\frac{h}{g}}$

(*c*) $t = \sqrt{\frac{2h}{3g}}$ (*d*) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$

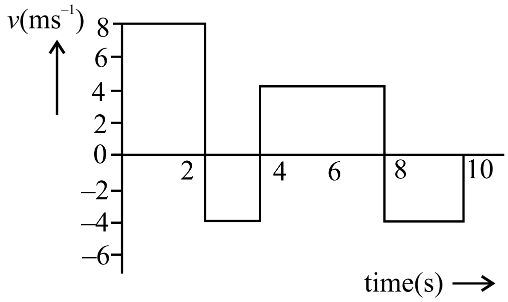
20. A tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height *h*/2. The velocity versus height of the ball during its motion may be represented graphically by (graph are drawn schematically and on not to scale). **[4 Sep, 2020 (Shift-I)]**



21. A ball is dropped from the top of a 100 m high tower on a planet. In the last 1/2 s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is _________.
[8 Jan, 2020 (Shift-II)]

GRAPHS

22. The velocity time graph of a body moving in a straight line is shown in figure.

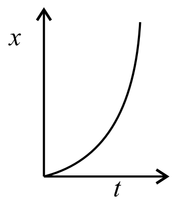
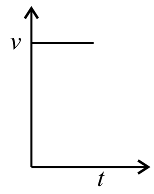
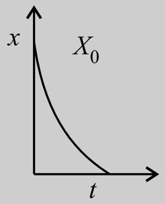
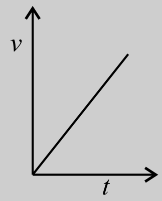
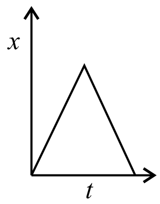
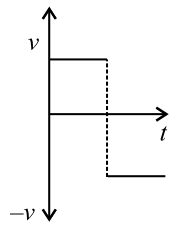
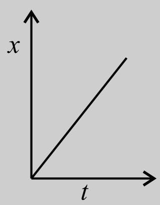
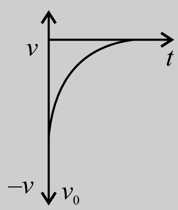


The ratio of displacement to distance travelled by the body in time 0 to 10 s is **[24 Jan, 2023 (Shift-II)]**

(*a*) 1 : 1 (*b*) 1 : 4

(*c*) 1 : 2 (*d*) 1 : 3

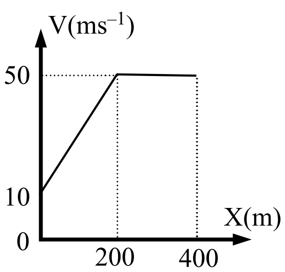
23. Match Column-I with Column-II:

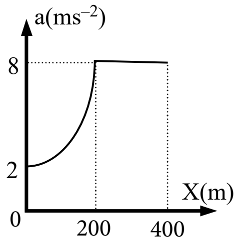
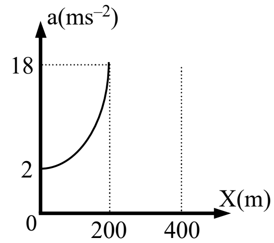
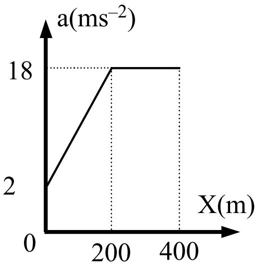
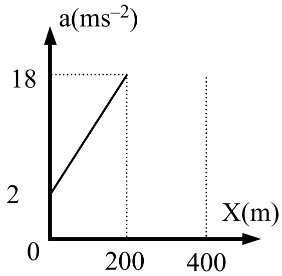
Column-I (x-t graphs)		Column-II (v-t graphs)	
A.		I.	
B.		II.	
C.		III.	
D.		IV.	

Choose the correct answer from the options given below:
[30 Jan, 2023 (Shift-I)]

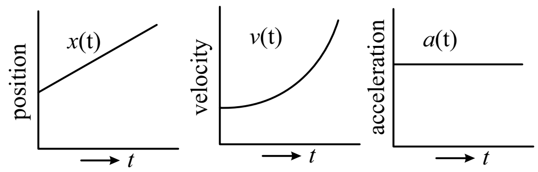
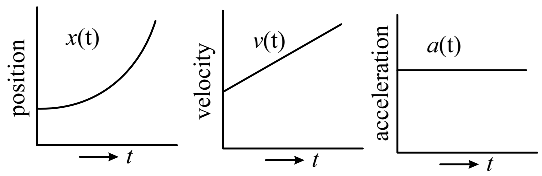
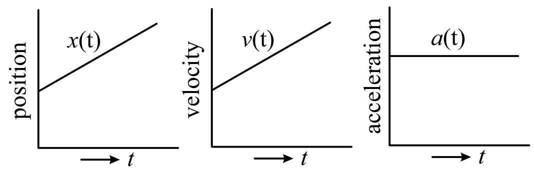
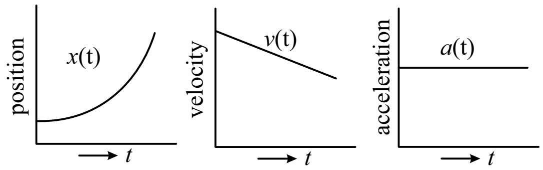
(*a*) A- II, B-IV, C-III, D-I
(*b*) A- I, B-II, C-III, D-IV
(*c*) A- II, B-III, C-IV, D-I
(*d*) A- I, B-III, C-IV, D-II

24. The velocity- displacement graph describing the motion of a bicycle is shown in the figure. The acceleration-displacement graph of the bicycle's motion is best described by:
[16 March, 2021 (Shift-I)]

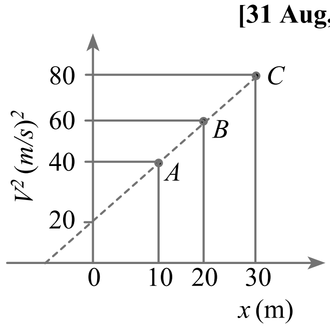


(*a*) 
(*b*) 
(*c*) 
(*d*) 

25. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by
[18 March, 2021 (Shift-I)]

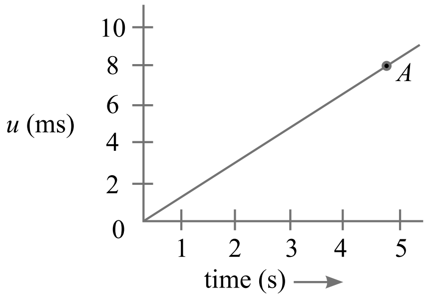
(*a*) 
(*b*) 
(*c*) 
(*d*) 

26. A particle is moving with constant acceleration '*a*'. Following graph shows v^2 versus x (displacement) plot. The acceleration of the particle is _____m/s^2.
[31 Aug, 2021 (Shift-II)]



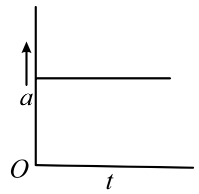
27. The speed versus time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval $t = 0$ to $t = 5$ s will be ___________.

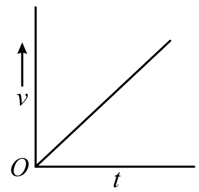
[3 Sep, 2020 (Shift-II)]

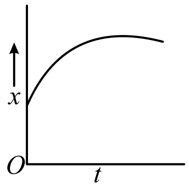


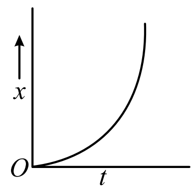
28. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify the figure that is not correctly representing the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)

[8 April, 2019 (Shift-II)]

(A) 

(B) 

(C) 

(D) 

(*a*) (A), (B), (C) (*b*) (A)

(*c*) (C) (*d*) (B), (C)

VARIABLE ACCELERATION

29. The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is: (Where v stands for velocity)

[25 July, 2021 (Shift-II)]

(*a*) $2n^2v^2$ (*b*) $2mnv^3$

(*c*) $2mv^3$ (*d*) $2nv^3$

30. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is __________.

[9 Jan, 2020 (Shift-I)]

PW CHALLENGERS

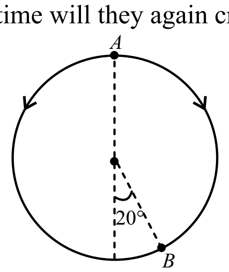
SINGLE CORRECT TYPE QUESTIONS

1. A man left his dog sitting on a large slippery ground and walks away with a velocity of 2m/s. When he is 207 m away from the dog, the dog decides to catch him and thereafter they move together. The dog can not develop acceleration more that 2m/s^2 in any direction due to slippery ground. If the maximum velocity with which dog can move is 16 m/s then the minimum time in which dog will meet the man is

(*a*) 20 sec (*b*) 21 sec

(*c*) 22 sec (*d*) 24 sec

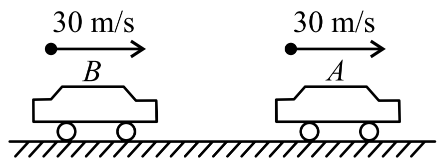
2. Two motorcyclists simultaneously start a race with constant speeds from point A to traverse on a circular track, one clockwise and other in anticlockwise sense. They simultaneously cross point B first time after a time interval of 10 minutes. If they continue to move, how long after they cross at B first time will they again cross at point B.



(*a*) 10 min (*b*) 95 min

(*c*) 90 min (*d*) 85 min

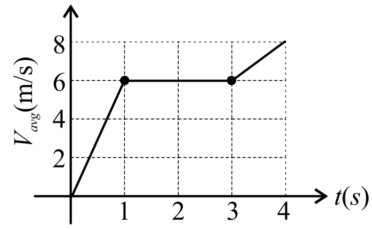
3. Two cars A and B are running on a highway with same velocity of 30 m/s. On application of brakes car A retards at a rate of 3 m/s^2 while car B retards at a rate of 4 m/s^2. Car A is running ahead of car B. In an emergency when driver of front car A applies brakes, in response the driver of near car B has to apply brakes to avoid accident. The response time of driver of car B is 1 sec. The minimum distance between them to avoid accident is

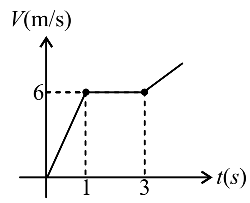


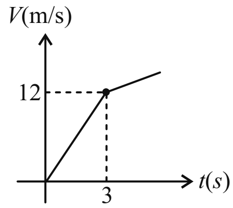
(*a*) 7.5 m (*b*) 6 m

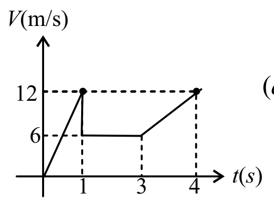
(*c*) 8.5 m (*d*) 10 m

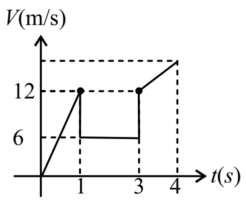
4. The relation between average velocity v_{av} of a particle and time t is shown in the graph. If during the time interval considered, the particle did not change its direction of motion, then plot instantaneous velocity as a function of time?



(*a*) 

(*b*) 

(*c*) 

(*d*) 

5. When a deer was 48 m from a leopard, the leopard starts chasing the deer and the deer immediately starts running away from the leopard with constant velocity. A leopard cannot run at high speeds for a long time and has to slow down due to fatigue. If we assume that the leopard starts with an initial speed of 30 m/s and reduces its speed in equal steps of 5 m/s after every 2 s interval, at what minimum speed must the deer run to escape from the leopard?

(*a*) 15 ms^{-1} (*b*) 16 ms^{-1} (*c*) 17 ms^{-1} (*d*) 18 ms^{-1}

6. At the initial instant, two particles are observed at different locations moving towards each other with velocities u_1 and u_2. If they are subjected to constant accelerations a_1, and a_2 in directions opposite to their initial velocities, they will meet twice. If time interval between these two meetings is Δt, find suitable expression for their initial separation.

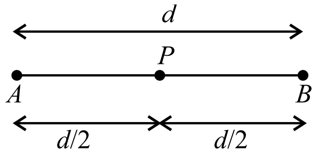
(*a*) $\dfrac{(u_1+u_2)^2}{a_1+a_2}+\dfrac{a_1+a_2}{8}(\Delta t)^2$

(*b*) $\dfrac{(u_1+u_2)^2}{2(a_1+a_2)}+\dfrac{a_1+a_2}{8}(\Delta t)^2$

(*c*) $\dfrac{(u_1+u_2)^2}{2(a_1+a_2)}-\dfrac{a_1+a_2}{8}(\Delta t)^2$

(*d*) $\dfrac{u_1^2+u_2^2}{2(a_1+a_2)}-\dfrac{a_1+a_2}{8}(\Delta t)^2$

7. A particle covers distance of d with uniform acceleration between point A & B. If its average velocity is v_{avg}, what could be range of magnitude of its instantaneous velocity v' at $\dfrac{d}{2}$ from point A.



(*a*) $v_{avg} \le v' \le \sqrt{3}\, v_{avg}$

(*b*) $\dfrac{v_{avg}}{\sqrt{3}} \le v' \le \sqrt{3} v_{avg}$

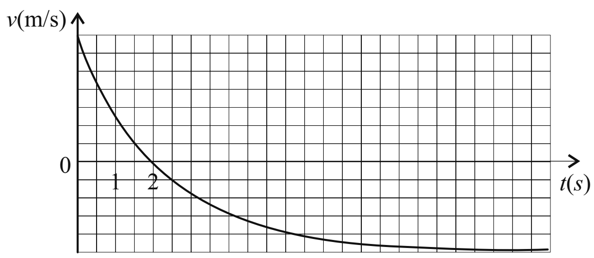
(*c*) $v_{avg} \le v' \le \sqrt{2}\, v_{avg}$

(*d*) $\dfrac{v_{avg}}{\sqrt{2}} \le v' \le \sqrt{2}\, v_{avg}$

8. Mr. Gupta used to walk to his office everyday and it took him 30 minutes. Once on his way he realized that he has forgotten to take his mobile phone. He knew that if he continued walking he will reach office 10 minutes before his office time. He went back home for the mobile phone, increased his speed by 20% and arrived the office 5 minutes late. What fraction of the way to office had he covered till the moment he decided to turn back?

(*a*) $\dfrac{7}{11}$ (*b*) $\dfrac{6}{11}$ (*c*) $\dfrac{5}{11}$ (*d*) $\dfrac{4}{11}$

9. To study effect of air resistance, a rubber ball was shot vertically upwards from a spring gun from 20th floor of a tall building. Velocity of the ball was recorded at regular intervals of time and the data obtained were plotted on a graph paper. Some of the marking on the axes are erased as shown in the following figure. With what speed did the ball strike the ground?



(*a*) 5 ms^{-1} (*b*) 10 ms^{-1} (*c*) 50 ms^{-1} (*d*) 25 ms^{-1}

10. Two motorboats that can move with velocities 4.0 m/s and 6.0 m/s relative to water are going up-stream in a river. When the faster boat overtakes the slower boat, a buoy is dropped from the slower boat. After lapse of a time interval, both the boats turn back simultaneously and move at the same speeds relative to the water as before. Their engines are switched off when they reach the buoy again. If the maximum separation between the boats is 200 m after the buoy is dropped and water flow velocity in the river is 1.5 m/s, find distance between the places where the faster boat passes by the buoy.

(*a*) 75 m (*b*) 300 m (*c*) 150 m (*d*) 350 m

Answer Key

CONCEPT APPLICATION

1. (24 km/hr, 24 km/hr) **2.** 10 km/h, $\frac{10}{3}\left(\sqrt{2}+1\right)$ km/hr **3.** **(***a***)** 2 m/sec^2, **(***b***)** 75 m **4.** (*a*) 1, (*b*) 125 m

5. (*b*) **6.** (*a*) **7.** (*c*) **8.** (*d*) **9.** (*a*) **10.** (*b*) **11.** (*c*) **12.** (*a*) **13.** (*b*)

PRARAMBH (TOPICWISE)

1. (*a*) **2.** (*c*) **3.** (*b*) **4.** (*d*) **5.** (*c*) **6.** (*b*) **7.** (*d*) **8.** (*c*) **9.** (*b*) **10.** (*a*)
11. (*b*) **12.** (*b*) **13.** (*b*) **14.** (*a*) **15.** (*c*) **16.** (*c*) **17.** (*c*) **18.** (*c*) **19.** (*b*) **20.** (*d*)
21. (*c*) **22.** (*a*) **23.** (*b*) **24.** (*c*) **25.** (*d*) **26.** (*b*) **27.** (*d*) **28.** (*c*) **29.** (*d*) **30.** (*a*)
31. (*c*) **32.** (*d*) **33.** (*c*) **34.** (*b*) **35.** (*b*) **36.** (*d*) **37.** (*d*) **38.** (*c*) **39.** (*d*) **40.** (*d*)

PRABAL (JEE MAIN LEVEL)

1. (*b*) **2.** (*c*) **3.** (*d*) **4.** (*c*) **5.** (*d*) **6.** (*a*) **7.** (*b*) **8.** (*a*) **9.** (*d*) **10.** (*c*)
11. (*a*) **12.** (*c*) **13.** (*c*) **14.** (*a*) **15.** (*d*) **16.** (*b*) **17.** (*d*) **18.** (*b*) **19.** [1125] **20.** [225]
21. [72] **22.** [24] **23.** [80] **24.** [7] **25.** [750]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*b*,*c*,*d*) **2.** (*c*,*d*) **3.** (*a*,*b*,*c*,*d*) **4.** (*a*,*c*,*d*) **5.** (*a,b,c,d*) **6.** (*a*,*d*) **7.** (*a*,*b*,*c*) **8.** (*c*) **9.** (*b*,*c*,*d*) **10.** (*b*)
11. (*c*) **12.** (*d*) **13.** (*c*) **14.** (*b*) **15.** (*a*) **16.** (*b*) **17.** (*d*) **18.** [0.82] **19.** [1.25]
20. [160.00] **21.** [30.00] **22.** [55] **23.** [15] **24.** [25] **25.** (*b*) **26.** (*d*) **27.** (*d*) **28.** (*c*) **29.** (*a*)

PYQ's (PAST YEAR QUESTIONS)

1. (*d*) **2.** [50] **3.** (*d*) **4.** [18] **5.** [3] **6.** (*b*) **7.** (*d*) **8.** [200] **9.** (*d*) **10.** [100]
11. (*b*) **12.** (*c*) **13.** (*b*) **14.** [120] **15.** (*d*) **16.** [392] **17.** [50] **18.** (*a*) **19.** (*a*) **20.** (*b*)
21. [8] **22.** (*d*) **23.** (*a*) **24.** (*d*) **25.** (*b*) **26.** [1] **27.** [20] **28.** (*c*) **29.** (*c*) **30.** [3]

PW CHALLENGERS

1. (*c*) **2.** (*c*) **3.** (*b*) **4.** (*d*) **5.** (*c*) **6.** (*c*) **7.** (*c*) **8.** (*d*) **9.** (*d*) **10.** (*b*)